Properly sampling a time-dependent signal

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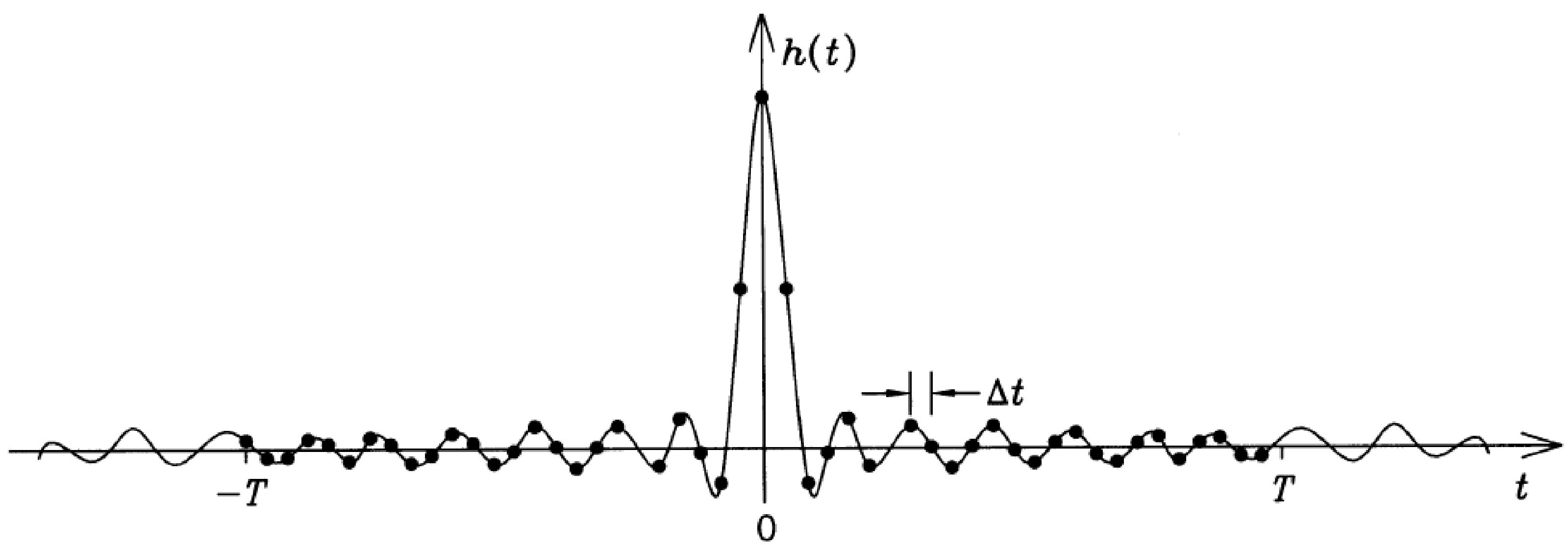
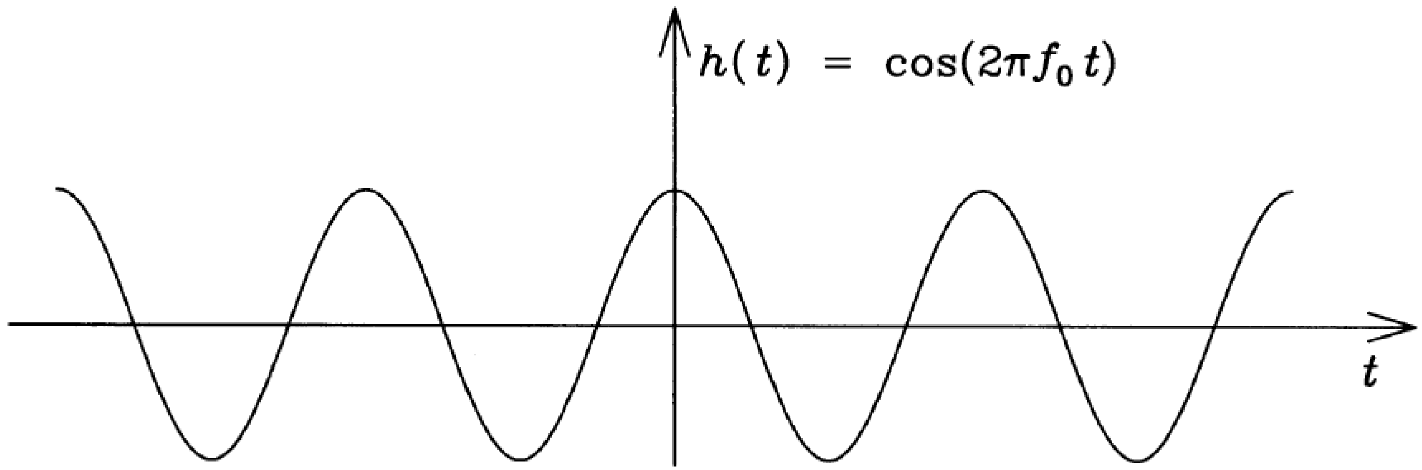
Consider measuring a signal as a function of time.

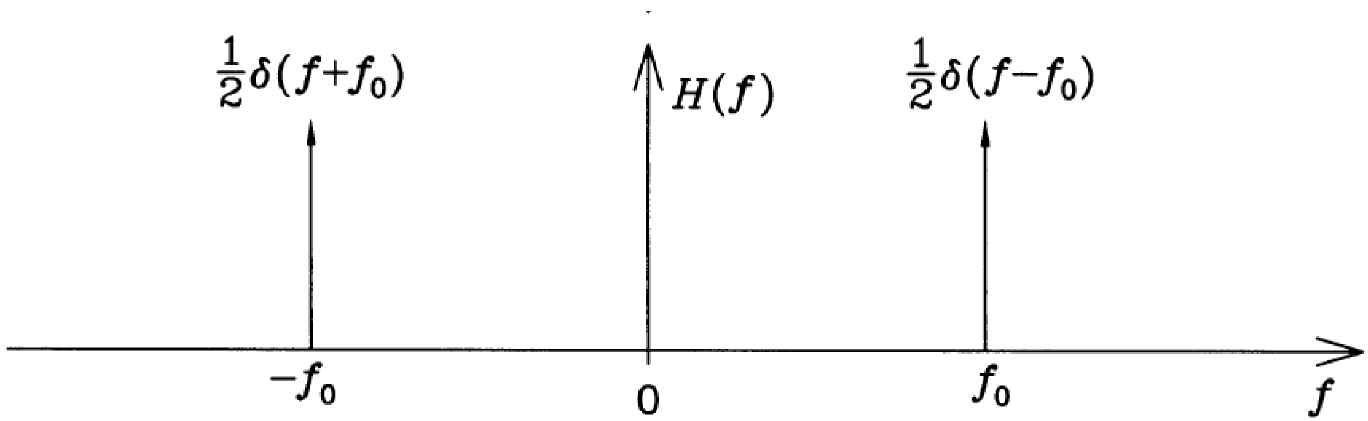
- **Sampling**: the measured signal will be sampled at the rate of $1/\Delta t$ hertz
- **Truncation**: the measured signal will be recorded over the time of $2T$ seconds

Improper sampling and truncation will cause the measured signal to have a different spectral content than the original signal.



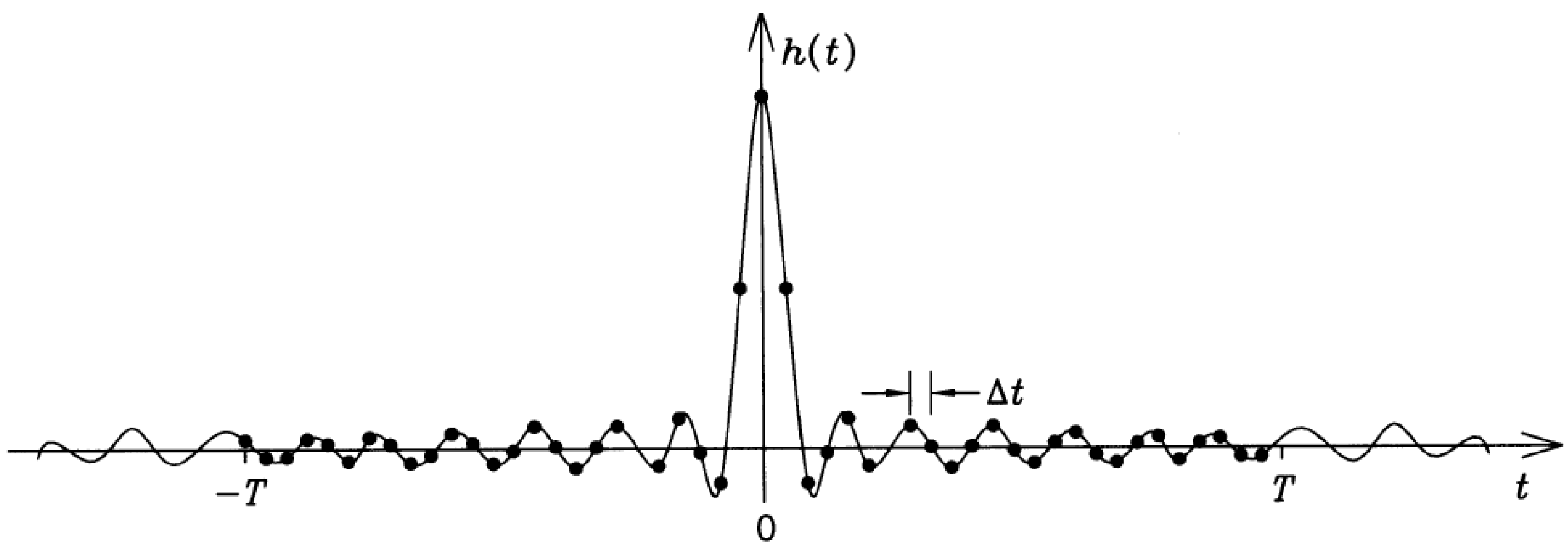
What is meant by the spectral content?
Consider the spectral content of a cosine



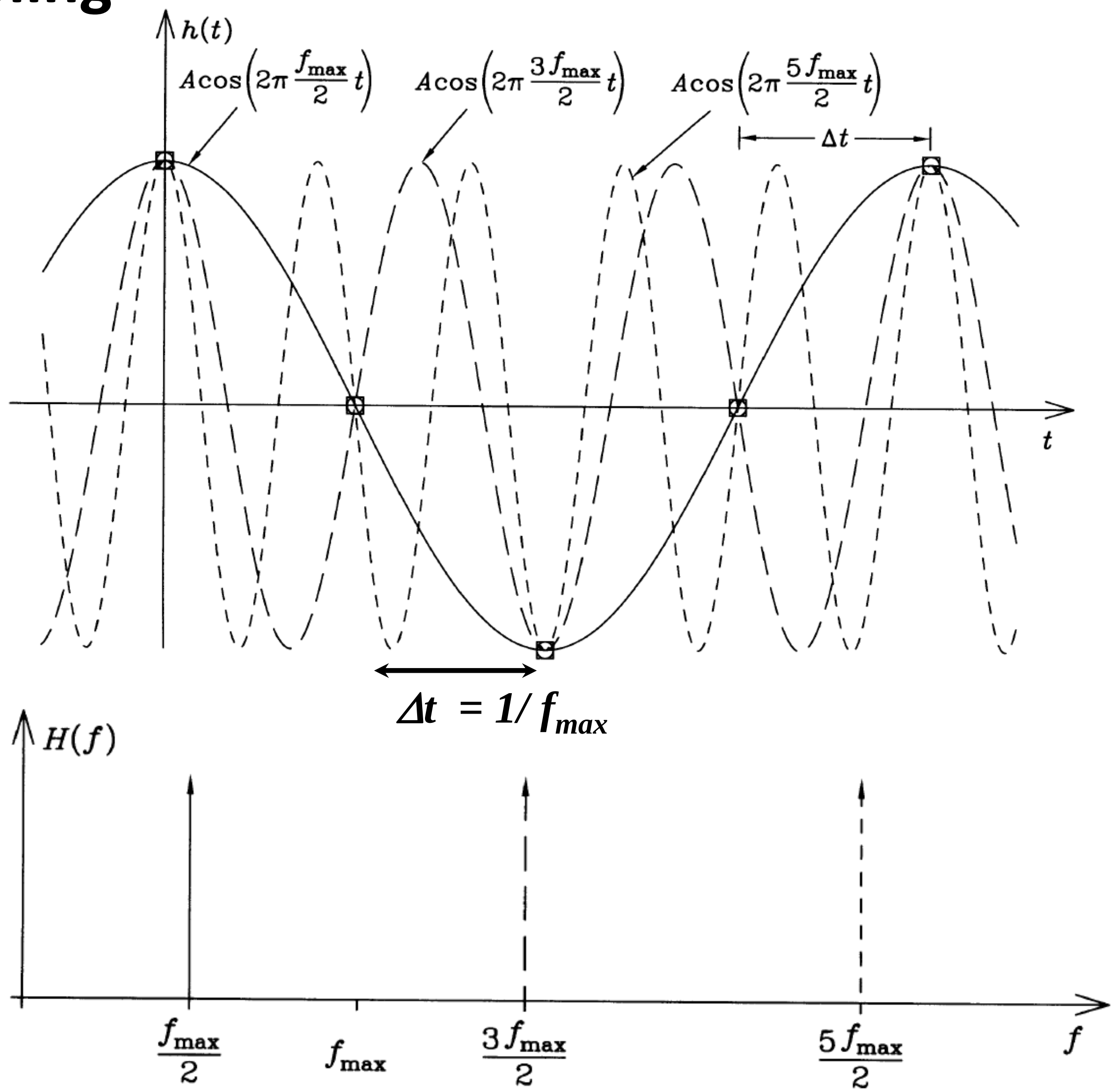


General Rules

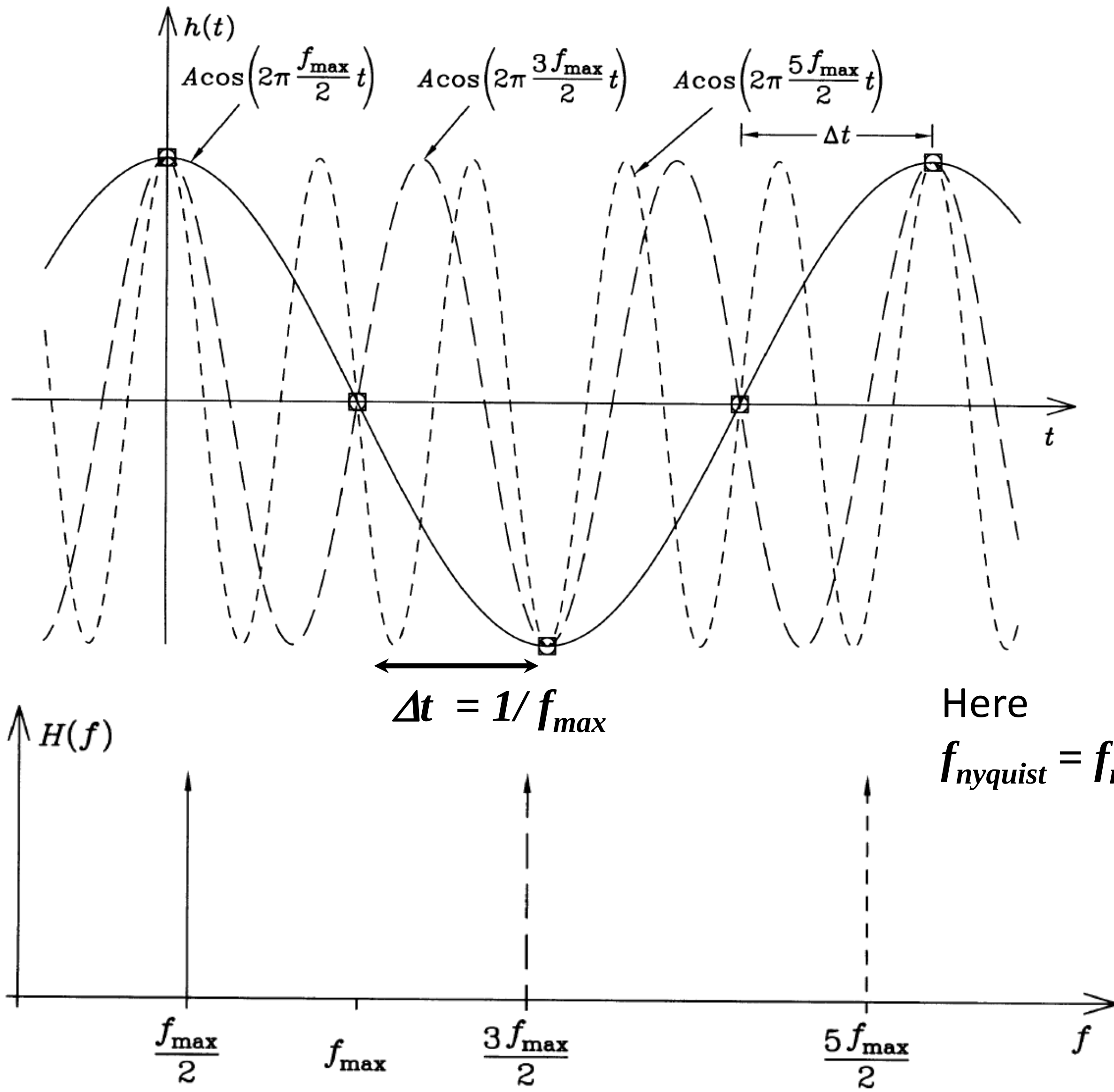
- For your signal determine highest frequency of interest f_{max}
- Nyquist Criteria: make sure that the sample rate is larger than twice the highest frequency of interest: $1/\Delta t > 2f_{max}$
- Make sure to take data for a time much longer than the time between points: $1/T << 1/(2\Delta t)$



Aliasing: creating false spectral components by improper signal sampling



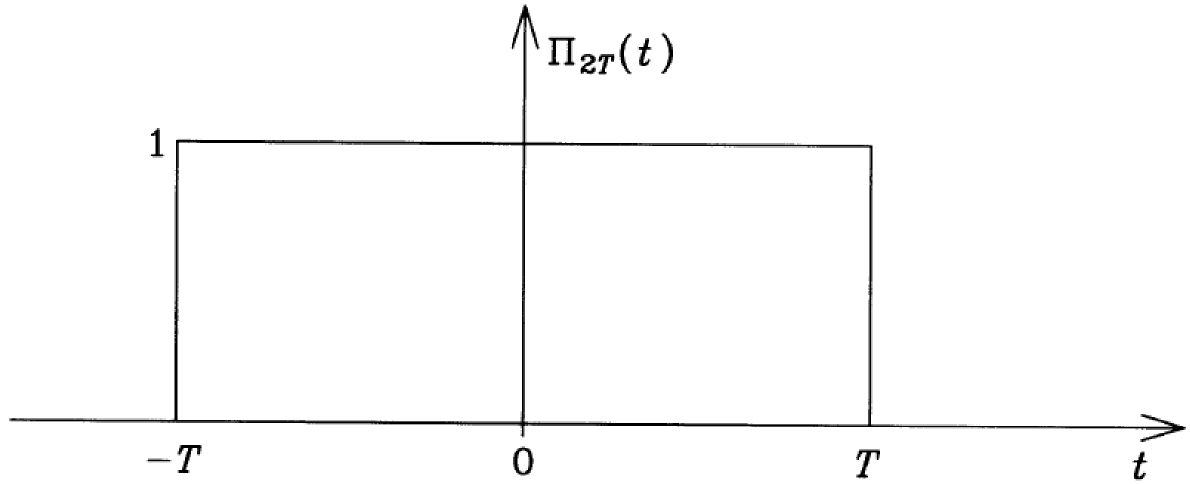
The Nyquist Frequency: to properly sample frequency *f* the sampling rate must be at least *2f*.

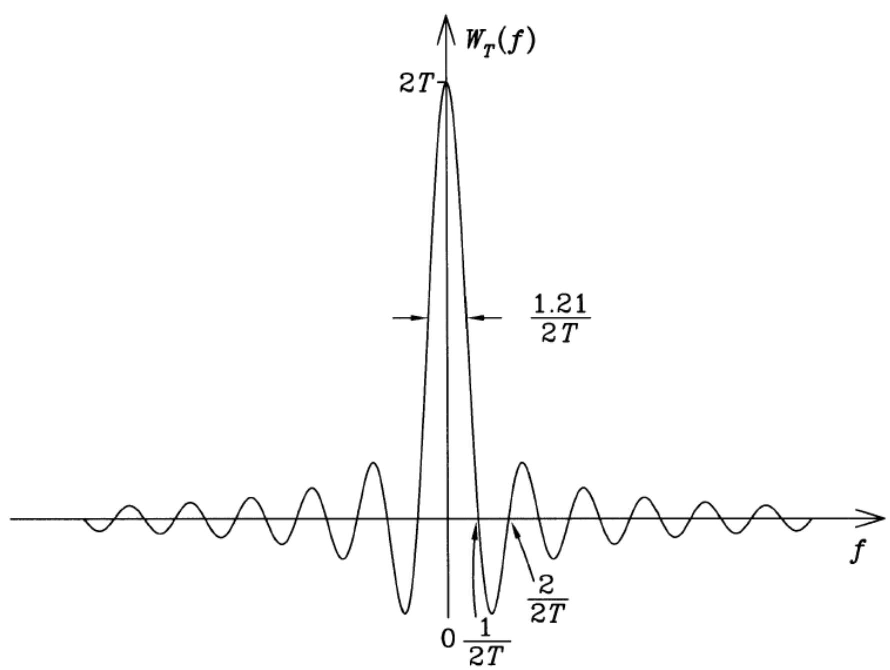


Here

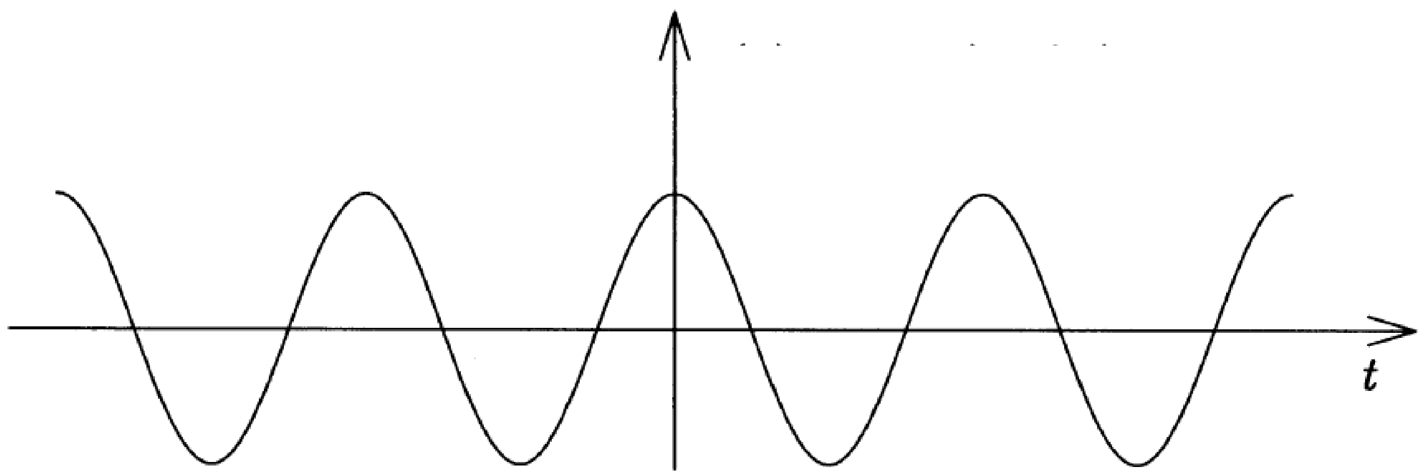
$f_{nyquist} = f_{max} = 2\ (f_{max}/2)$

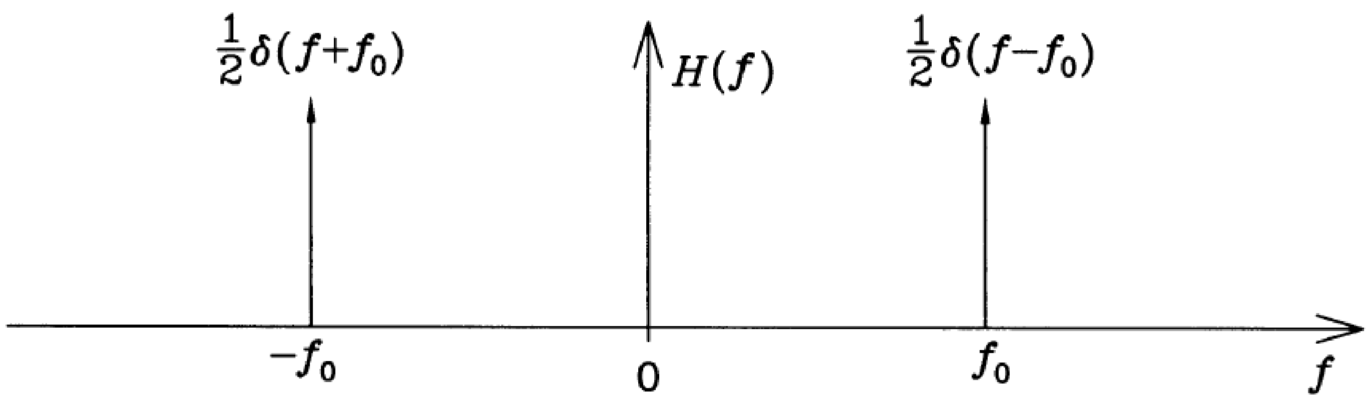
The effect of truncation:
multiplying signal by a boxcar function



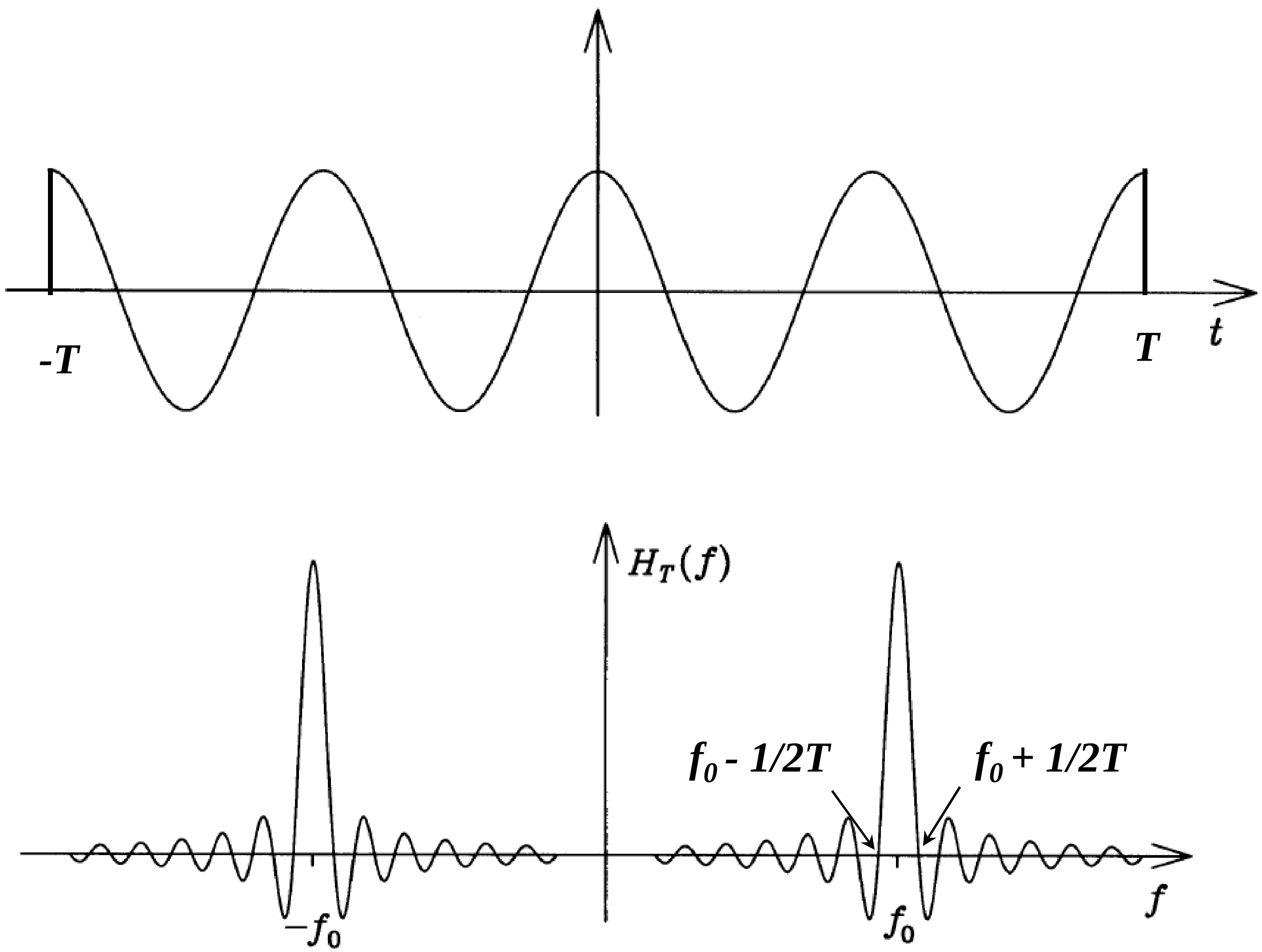


Cosine signal and its Fourier Transform

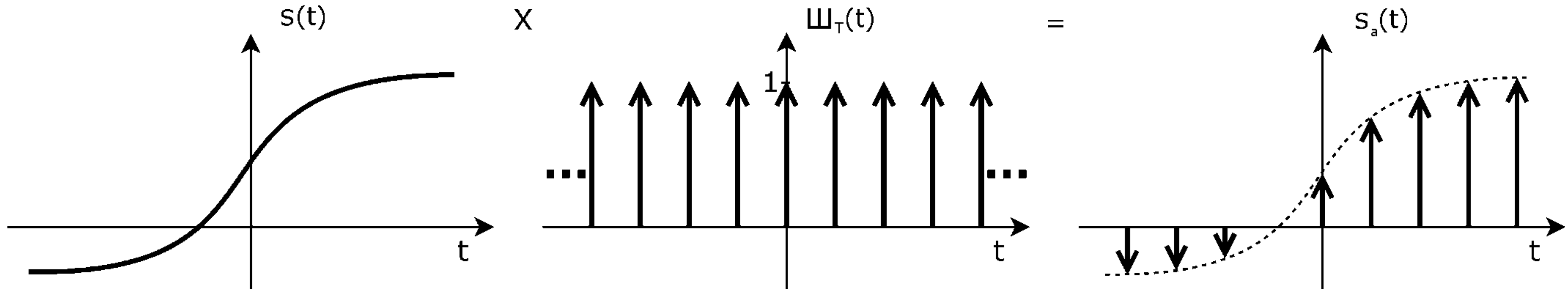




A truncated cosine and its Fourier transform

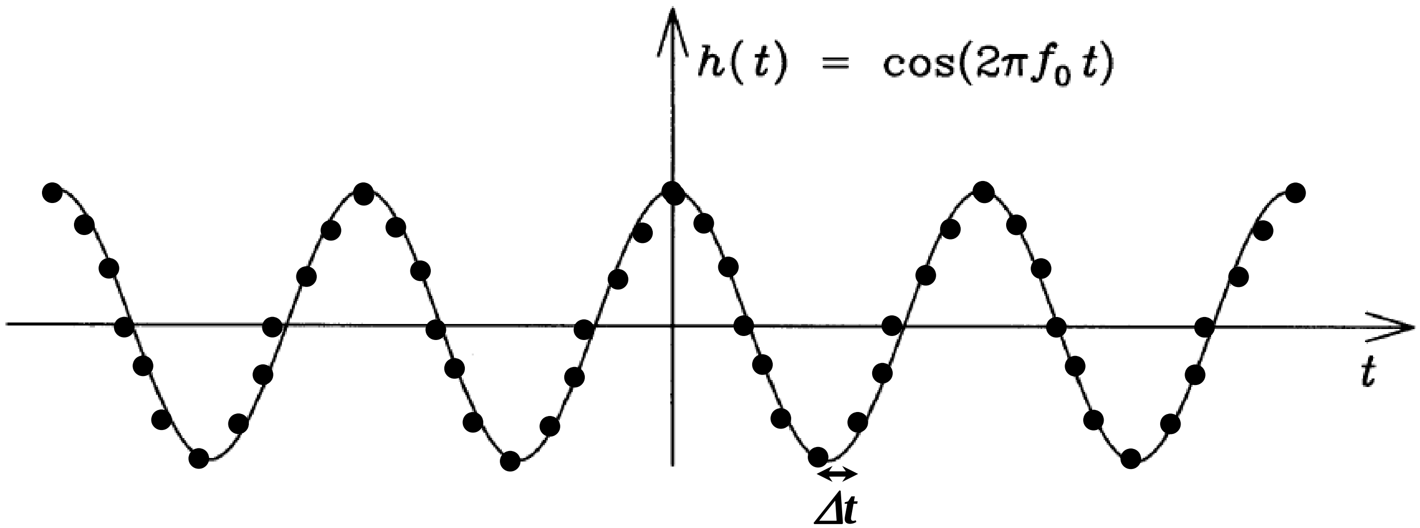


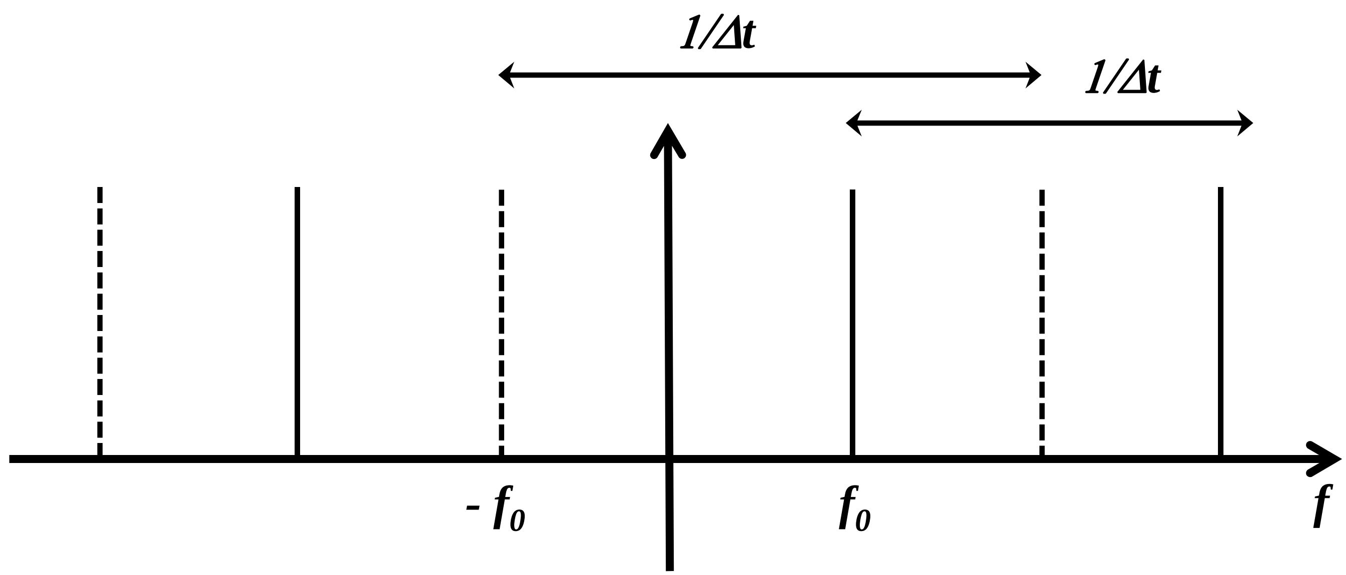
The effect of sampling at rate $1/\Delta t$:
Multiplying the actual signal by a Dirac comb, an infinite series of delta function



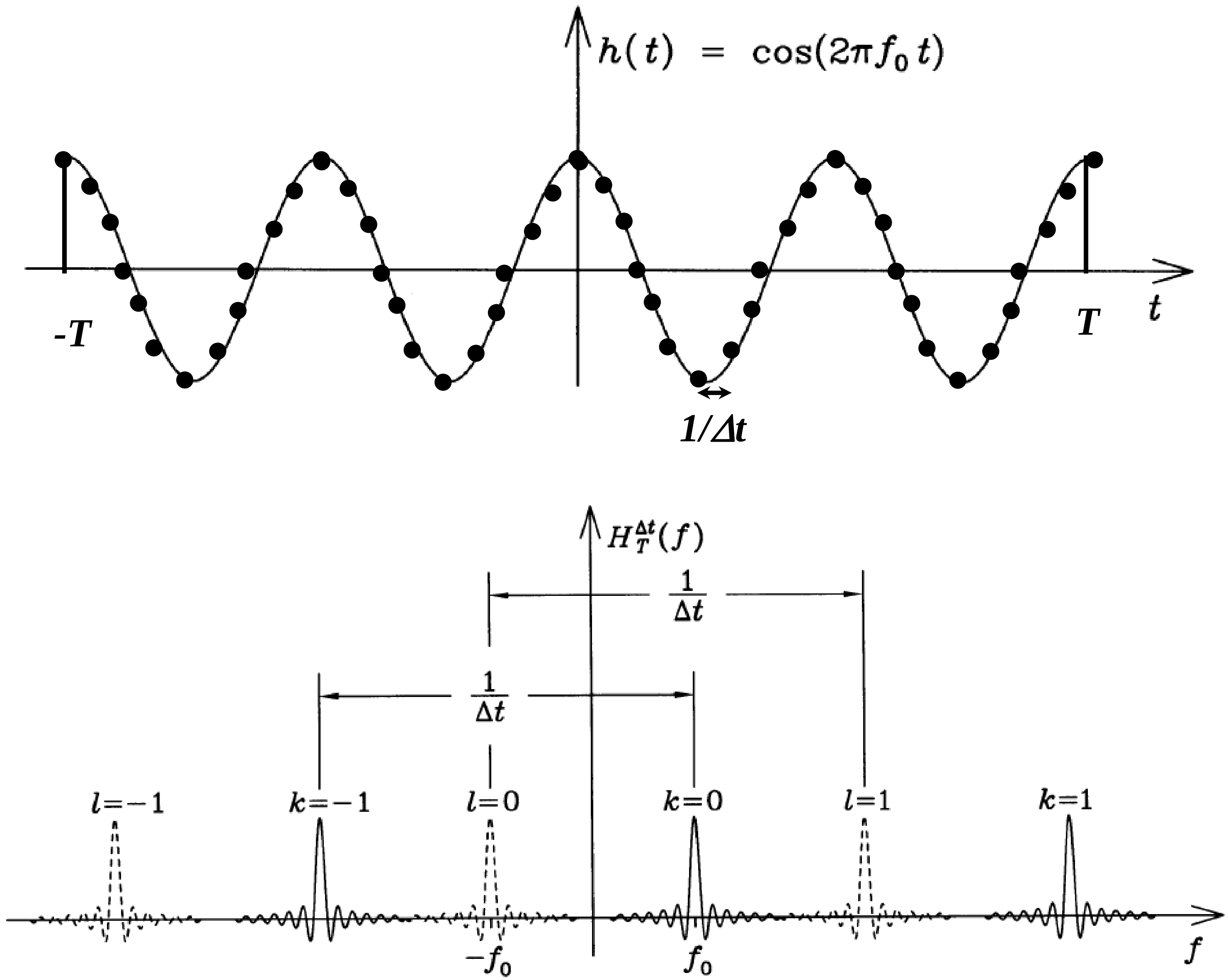
The Fourier transform of a Dirac comb times a signal will be the convolution of a Dirac comb spaced by $1/\Delta t$ and the Fourier transform of the signal

A sampled cosine and its Fourier Transform

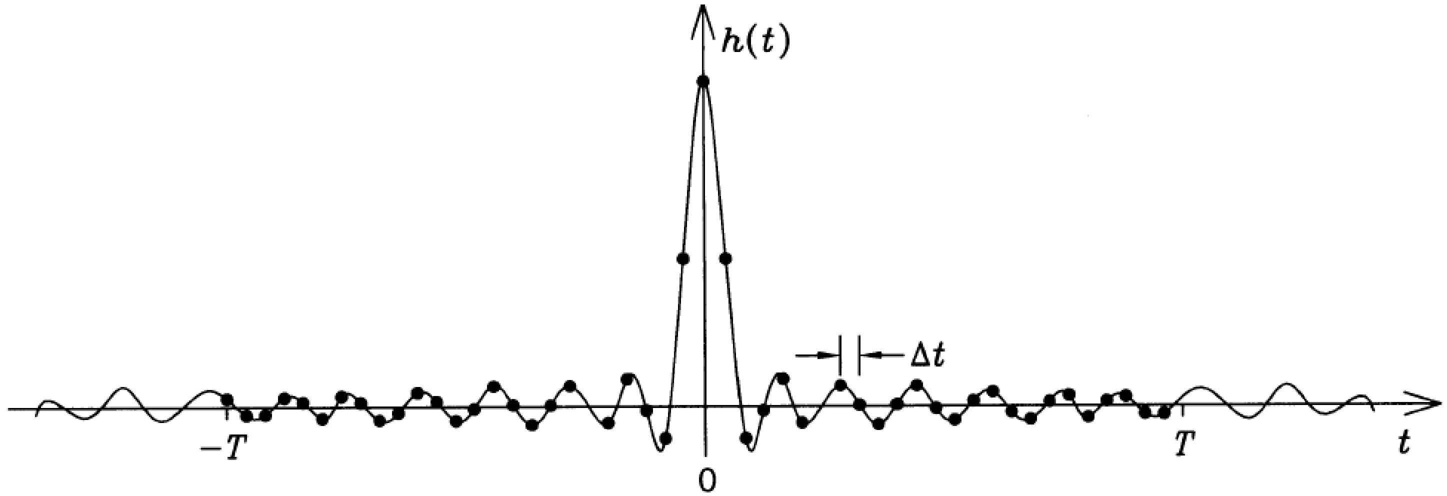




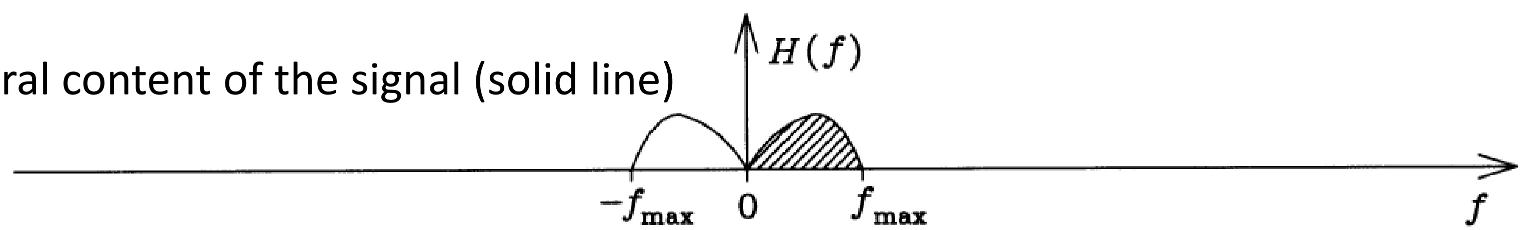
A truncated and sampled cosine and its Fourier Transform



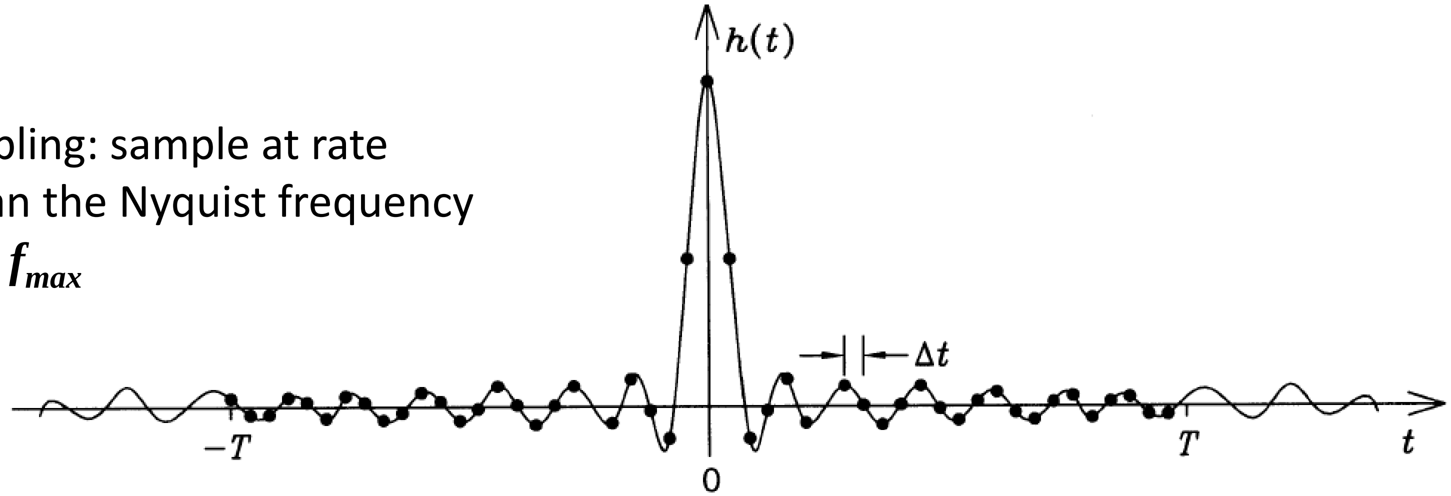
The effect of truncation and sampling a signal



Spectral content of the signal (solid line)



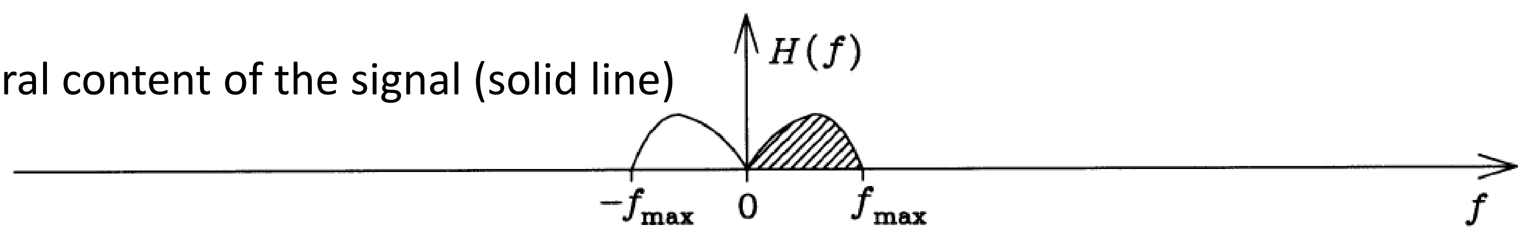
The effect of truncation and oversampling a signal



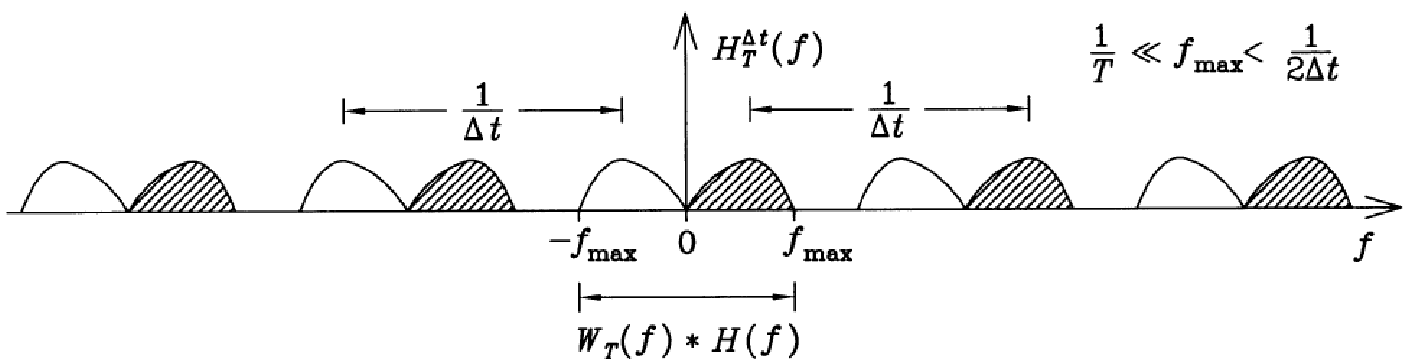
Oversampling: sample at rate larger than the Nyquist frequency

$1/\Delta t > 2\, f_{max}$

Spectral content of the signal (solid line)

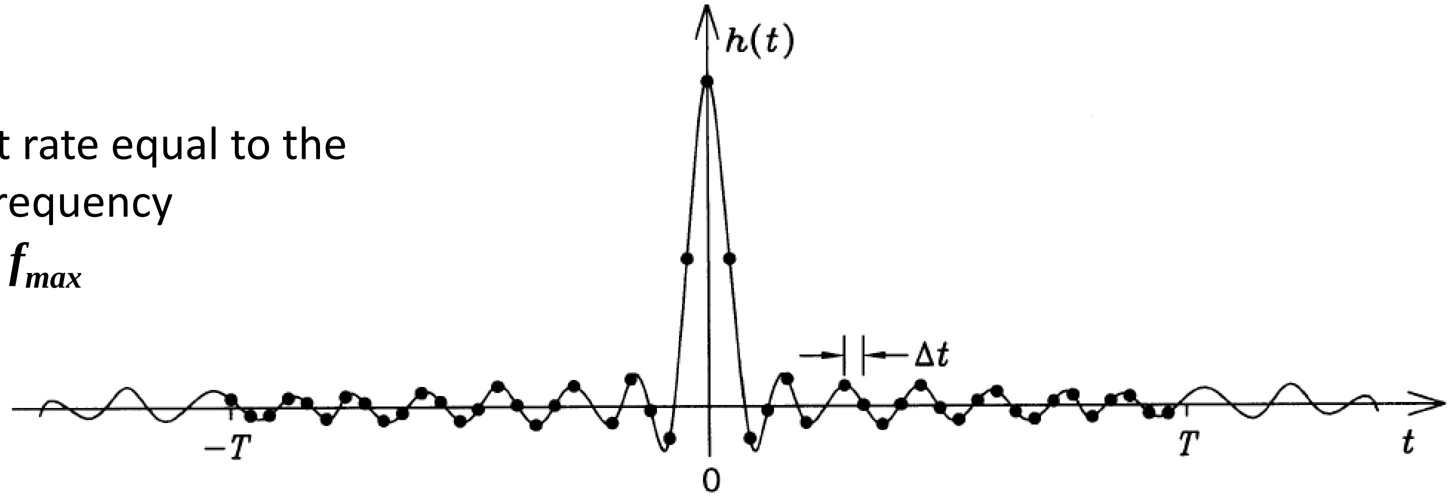


Truncated and oversampled sampled at $1/\Delta t > 2\, f_{max}$

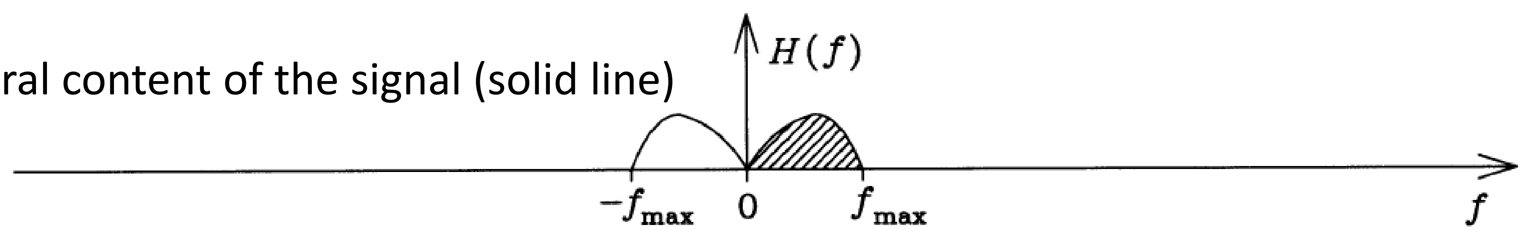


The effect of truncation and sampling at the Nyquist frequenc

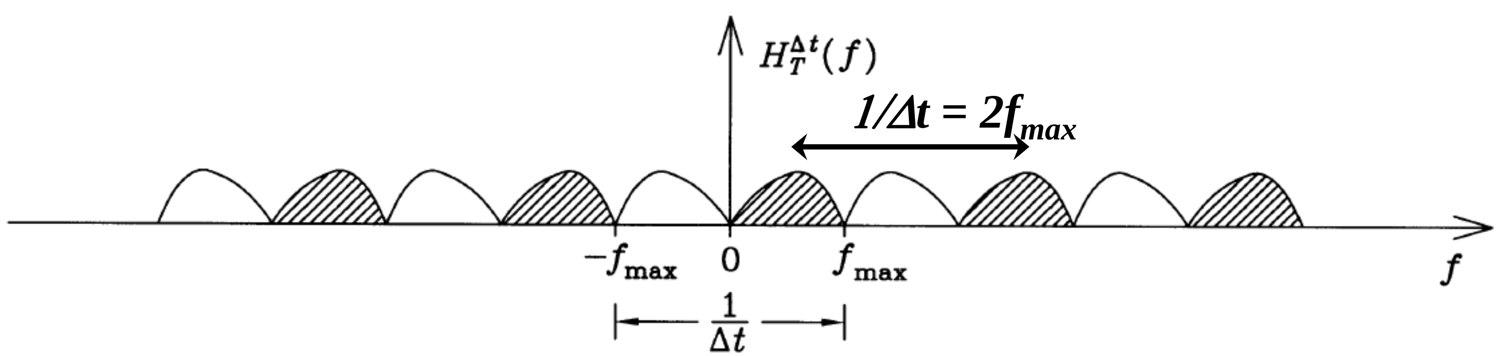
Sample at rate equal to the Nyquist frequency
$1/\Delta t = 2 f_{max}$



Spectral content of the signal (solid line)

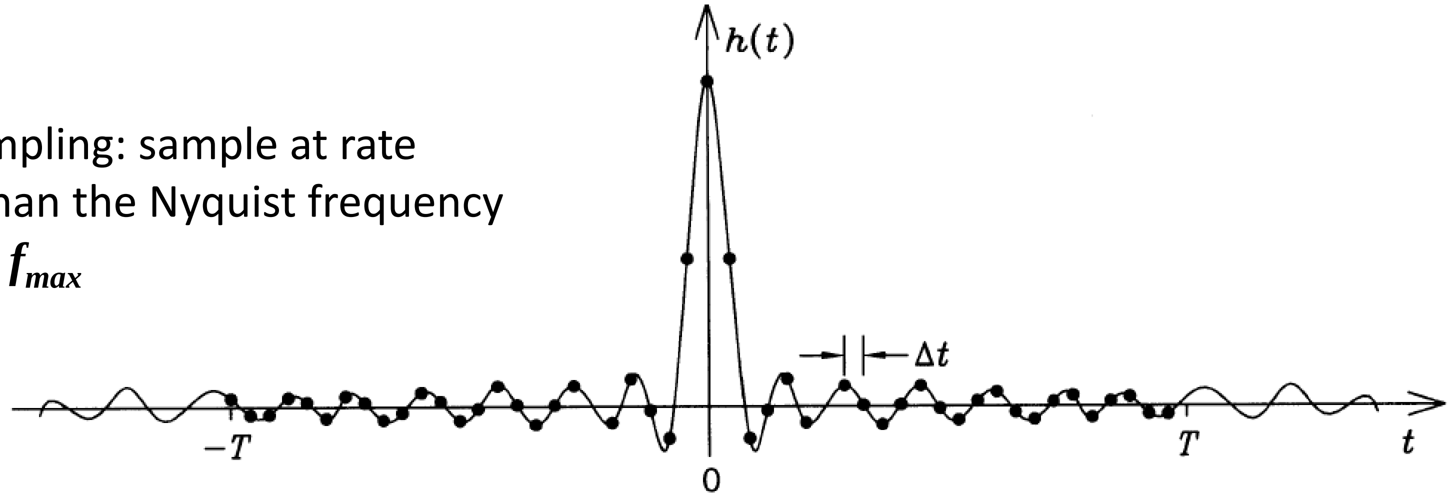


Truncated and sampled sampled at $2 f_{max}$

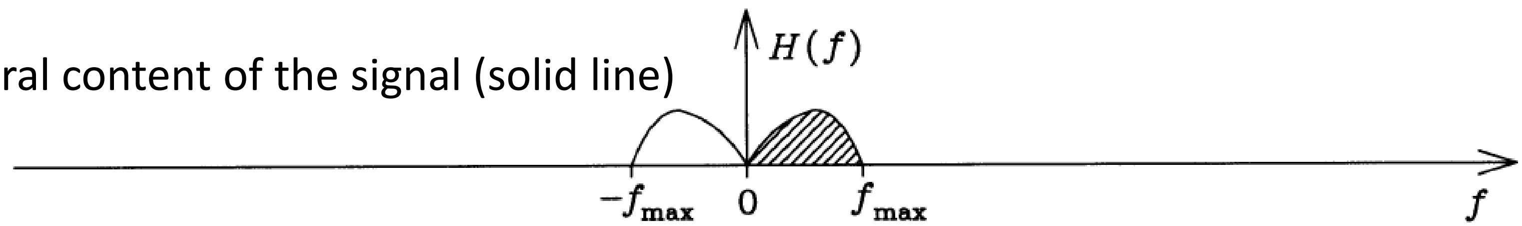


The effect of truncation and undersampling

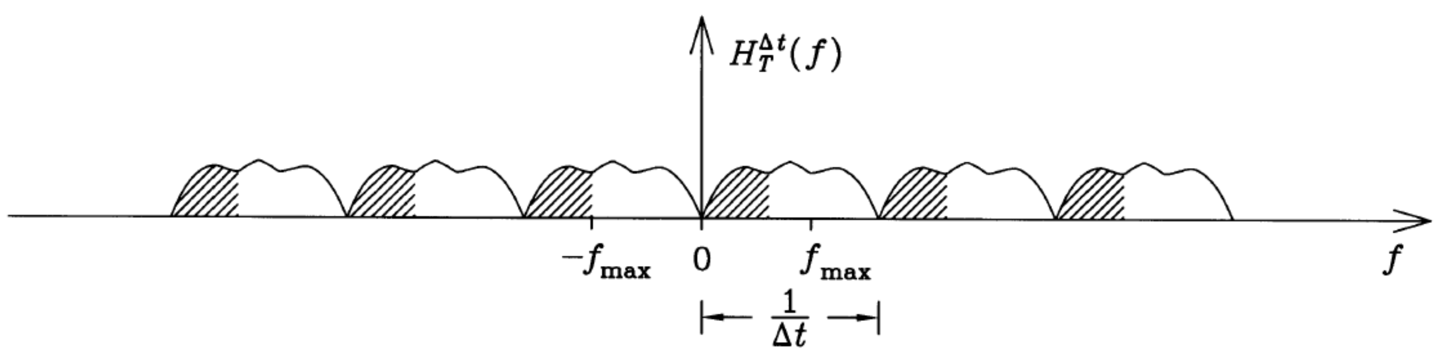
Undersampling: sample at rate smaller than the Nyquist frequency
$1/\Delta t < 2 f_{max}$



Spectral content of the signal (solid line)



Truncated and sampled sampled at $1/\Delta t < 2 f_{max}$



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Exercise

Use a Techtronic sampling oscilloscope to measure a sinusoidal signal of frequency f. Observe aliasing by choosing a time scale, and thus a sample rate that undersamples the data.

Sample Rate = Record Length/Time Duration