Three-Body Collisions in Ultracold Quantum Gases

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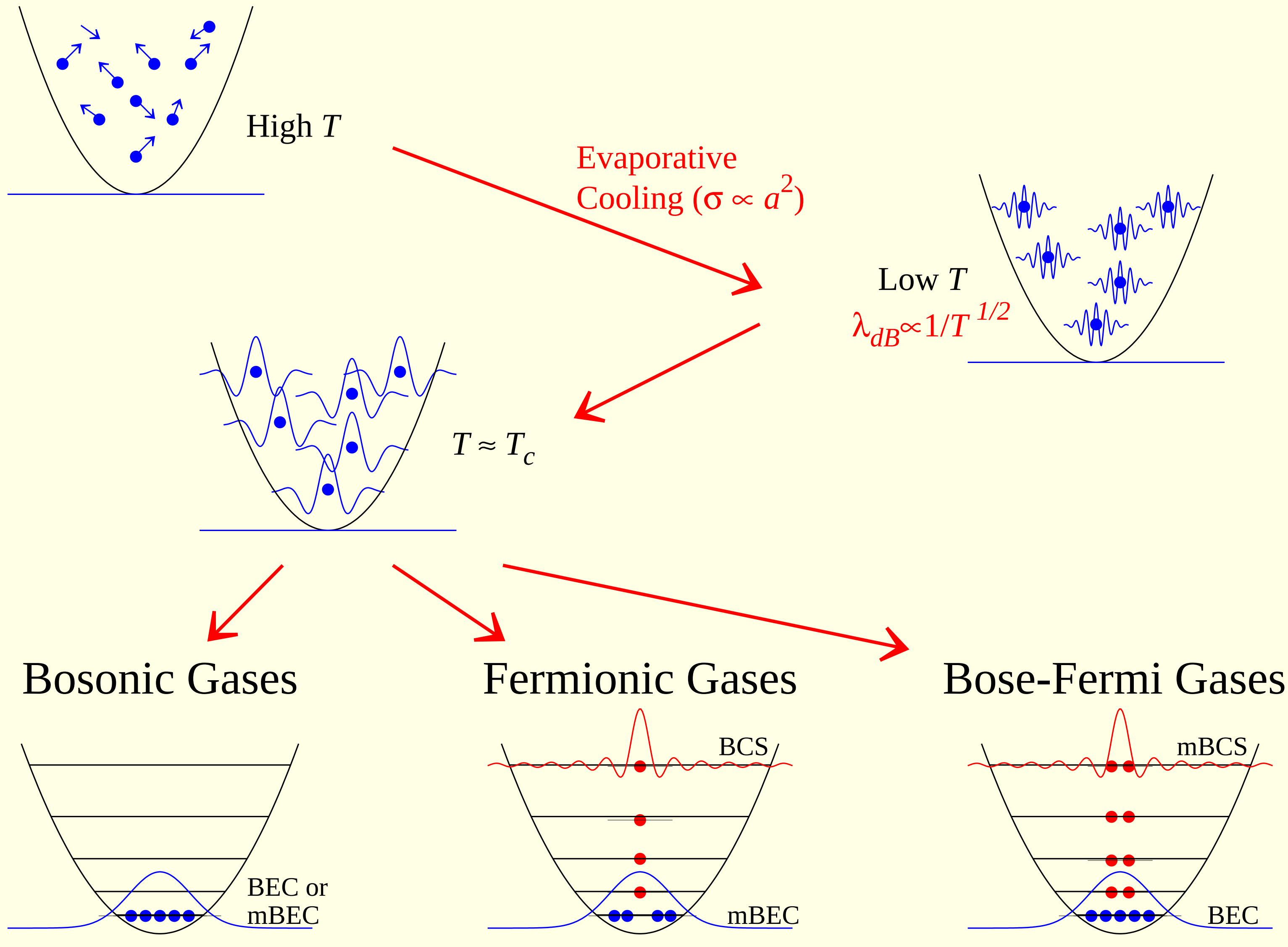
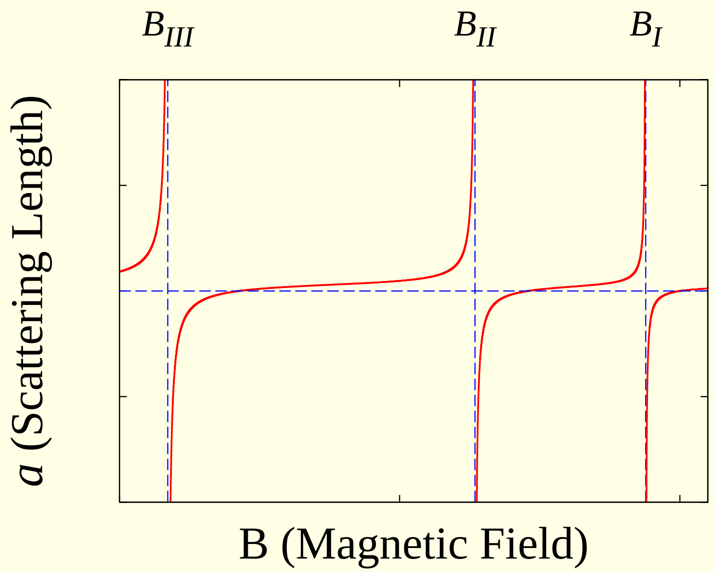
What's the big deal with ultracold temperatures ?



Experiments

External Magnetic Field: Feshbach Resonance

(Control the of interatomic interactions)



$a > 0 \rightarrow$ Effective Repulsive Interactions

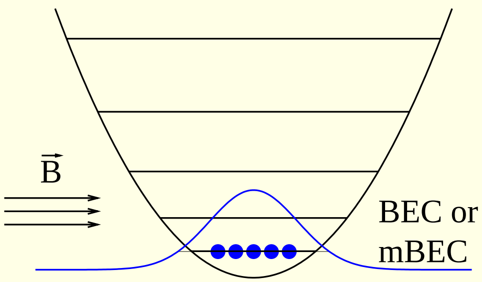
$a < 0 \rightarrow$ Effective Attractive Interactions

$|a| \gg r_0$: Strongly Interacting Regime !

(When Some Interesting Physics Start to Happen ...)

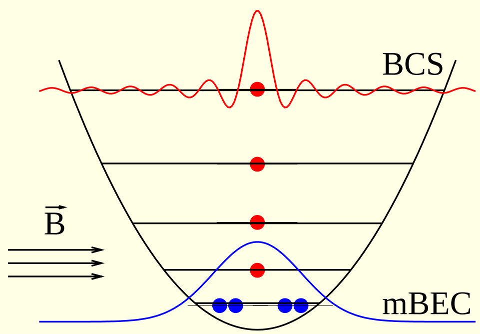
Ultracold Quantum Gases

Bosonic Gases



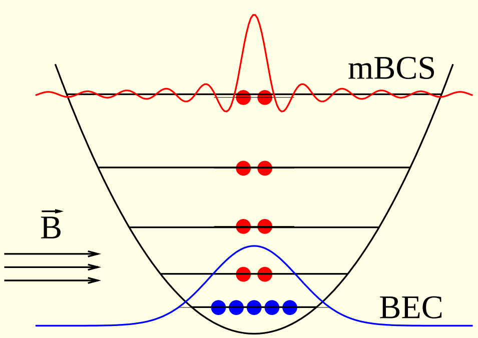
- Collapse (Bosanova)
- Atom Lasers
- Atom Chip

Fermionic Gases



- BCS (Superconductivity)
- Condensation of Cooper Pairs
- BEC-BCS crossover
- Long-Lived Molecules

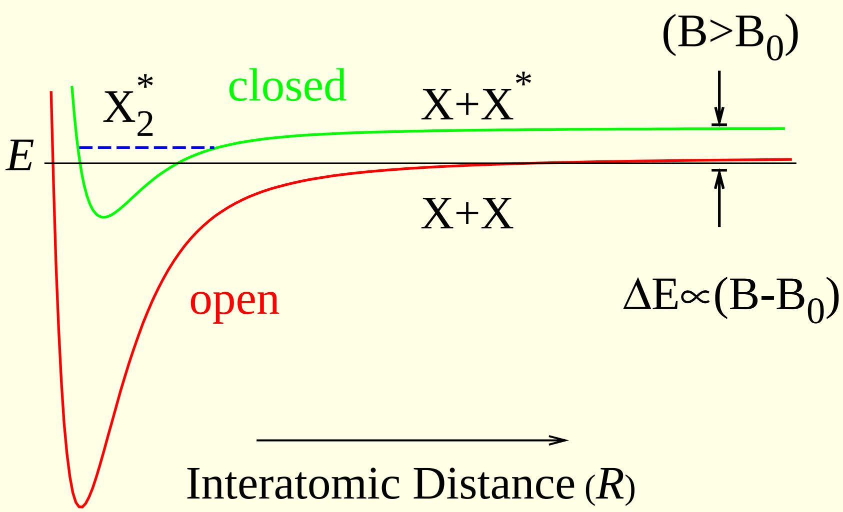
Bose-Fermi Gases



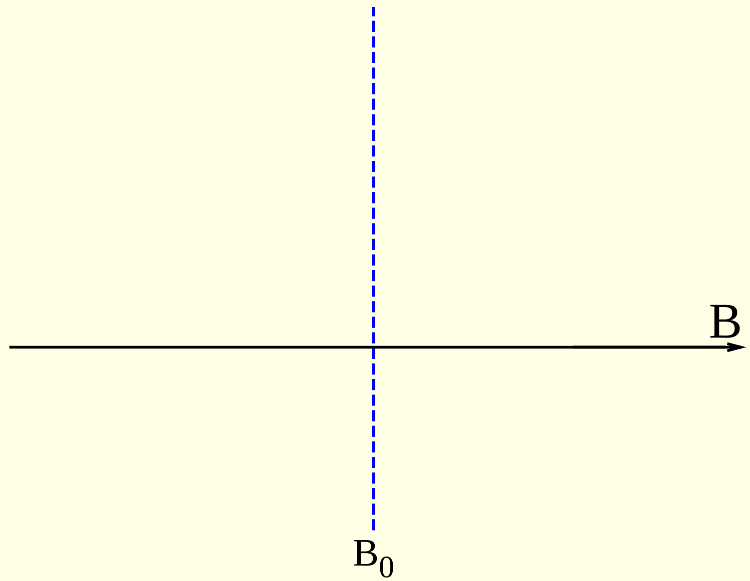
- mBCS/BEC : High T_c
- Boson Mediated Cooper Paring
- Ultracold Polar Molecules

Tuning Interatomic Interactions

Interatomic Potential



Scattering Length a



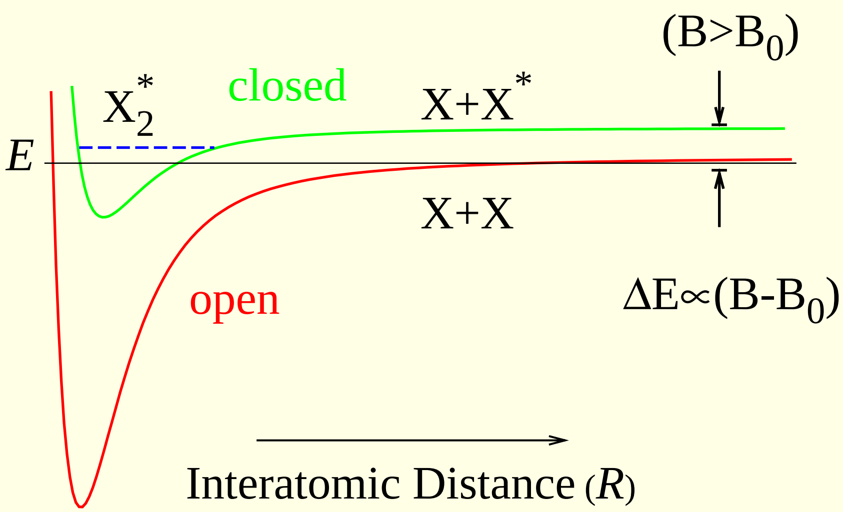
$$a = \lim_{k\to 0} \frac{\tan(\delta_o + \delta_c)}{k}$$

$\Rightarrow$

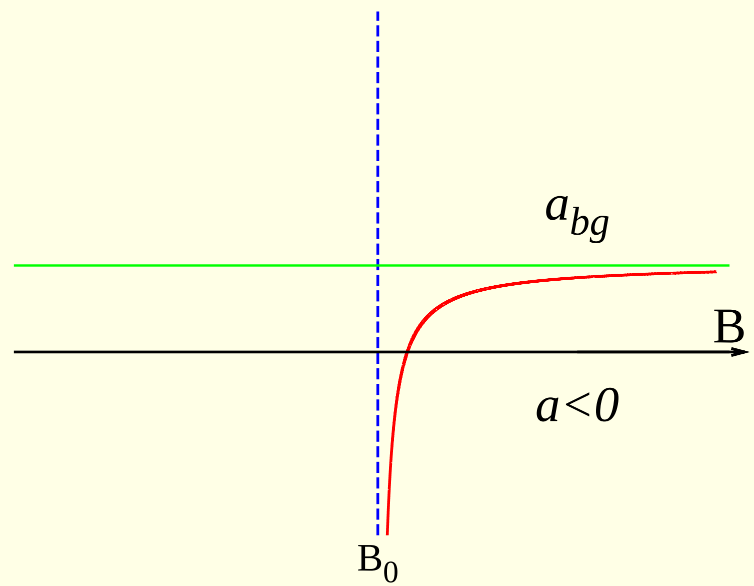
$$\delta_i \approx \int \sqrt{2\mu r^2[E - V(R)]dr}$$

Tuning Interatomic Interactions

Interatomic Potential



Scattering Length *a*



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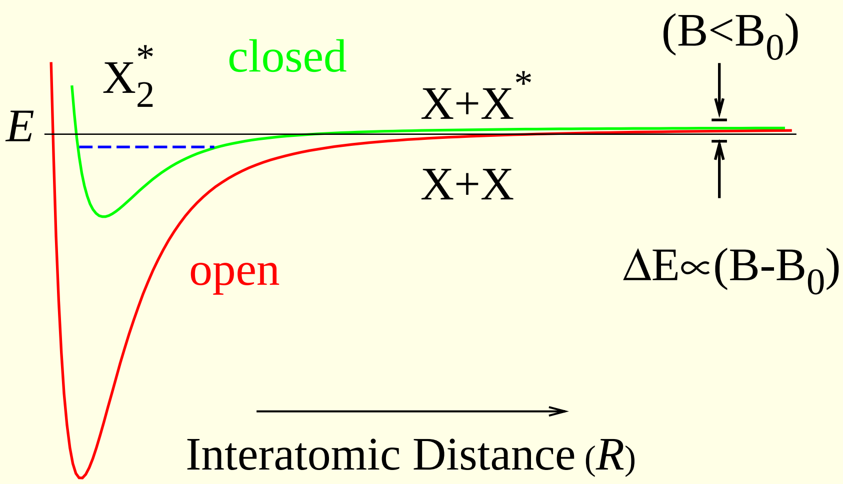
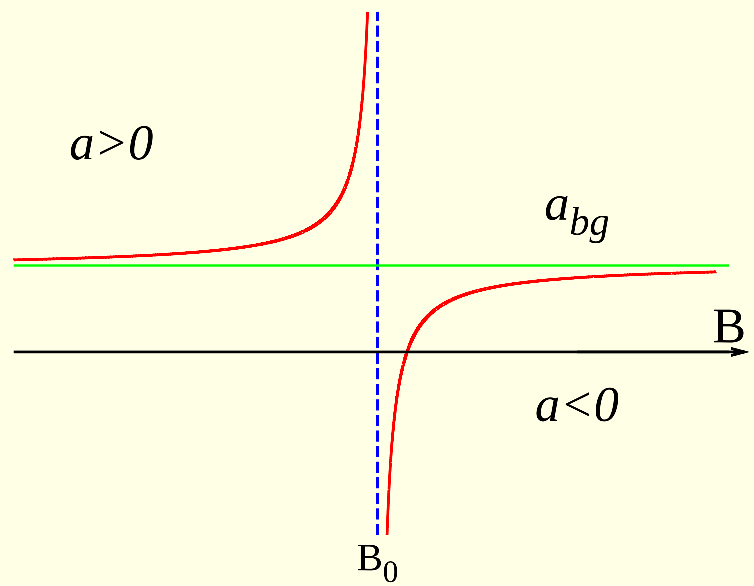
Tuning Interatomic Interactions

Interatomic Potential



Scattering Length a



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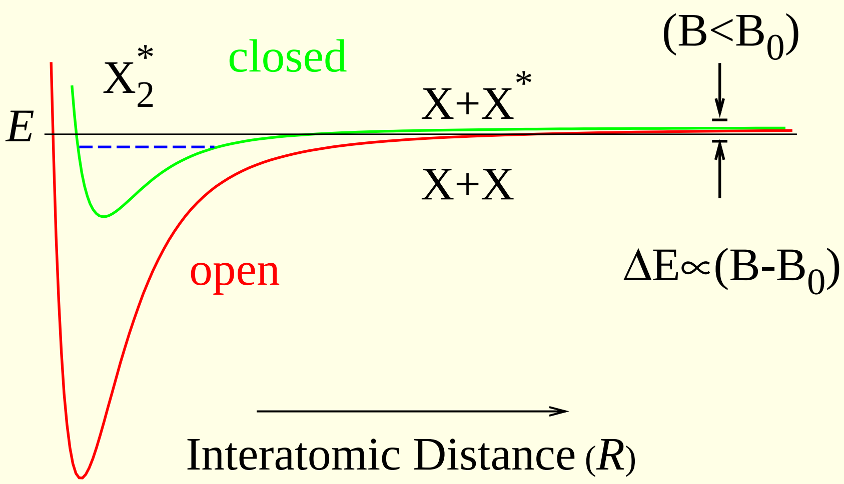
Interatomic Potential



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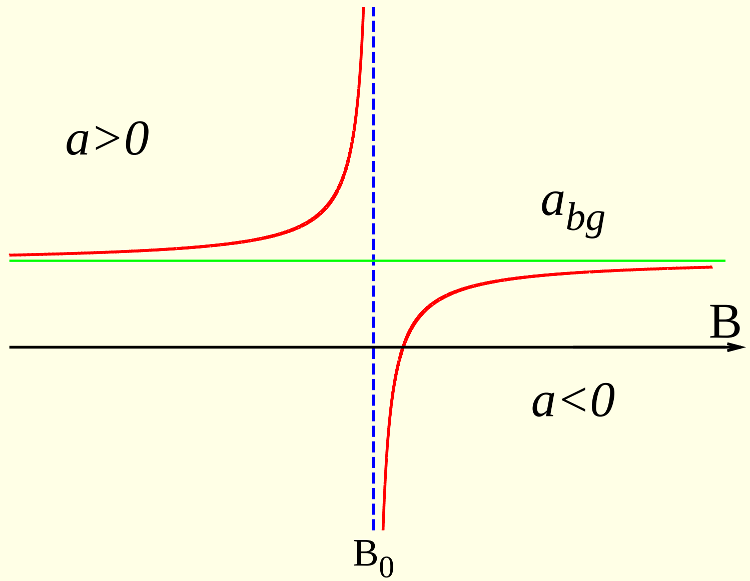
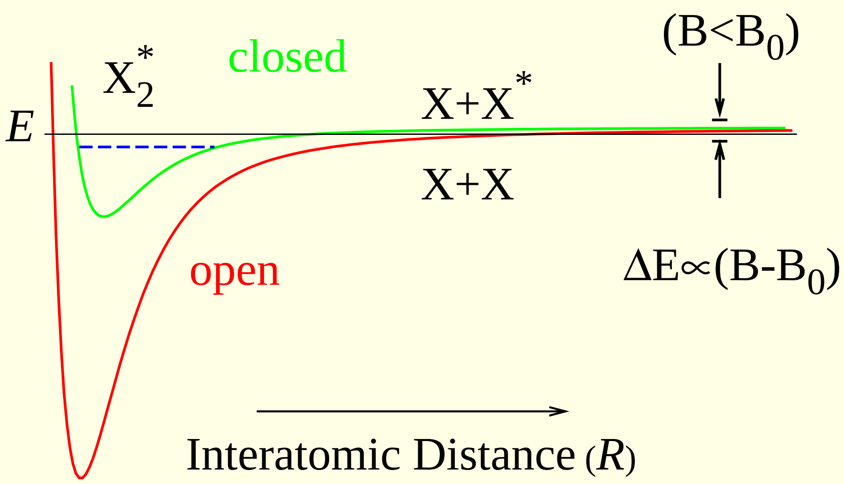


$a > 0$ → Repulsive Interactions (weakly bound molecules)

$a < 0$ → Attractive Interaction (deeply bound molecules)

Tuning Interatomic Interactions

Interatomic Potential



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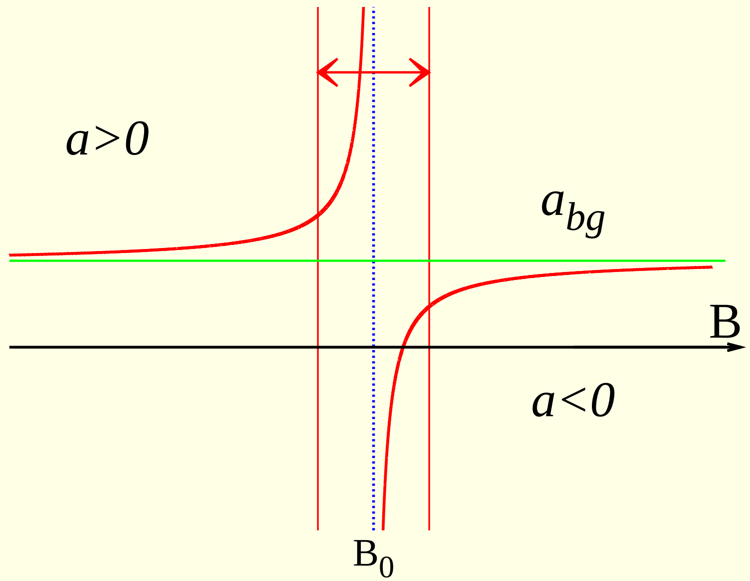

$|a| \gg r_0$: Strongly Interacting Regime !

(when interesting physics start to happen)

Losses Process in Ultracold Atomic Gases

Elastic Collisions ($\sigma \propto a^2$) are "good" : Evaporative Cooling !

Inelastic Collisions are "BAD" : Atomic and Molecular Losses

Rate Equation for the atomic density

$$\frac{d}{dt}n(t) = K_2 n^2(t) - K_3 n^3(t)$$

$K_2 \rightarrow$ Two-Body (negligiable or suppressed)

$K_3 \rightarrow$ Three-Body (Dominate !)

(Identical Bosons: $K_3 \propto a^4$!)

Losses Process in Ultracold Atomic Gases

Three-Body Recombination (K_3)

$$X + X + X \to X_2^* + X + E_{vl}$$

Collision Induced Dissociation (D_3)

$$X_2^* + X + E_{vl} \to X + X + X$$

Vibrational Relaxation (V_{rel})

$$X_2^* + X \to X_2 + X + E$$

Losses Process in Ultracold Atomic Gases

For $|a| \gg r_0$ and $T \to 0$: 3-Body Physics becomes Universal

(Because $\lambda \gg |a|$!)

$$L_3 = \Big\langle \text{Non-Universal Features} \Big\rangle \Big\langle \text{Universal Features} \Big\rangle$$

$\Rightarrow$ Non-Universal Features: Details of the interatomic interaction

$\Rightarrow$ Universal Features: E and a dependence

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$\Rightarrow$ Non-Universal Features: Details of the interatomic interaction

$\Rightarrow$ Universal Features: E and a dependence

Our Job: Extract the Universal aspects !

$\Rightarrow$ Threshold Laws

$\Rightarrow$ Scaling Laws

Losses Process in Ultracold Atomic Gases

What is Known ?

	J^π	V_{rel}			K_3 (D_3)		
		E	$a>0$	$a<0$	E	$a>0$	$a<0$
BBB	0^+	**const**	$\boldsymbol{a}^{\triangle}$	?	**const** $(\boldsymbol{k^4})$	$\boldsymbol{a^4}^{\triangle}$	$\boldsymbol{\lvert a\rvert^4}^{\triangle}$
	1^-	k^2	?	?	k^6 (k^{10})	?	?
	2^+	k^4	?	?	k^4 (k^8)	$a^{8\blacktriangle}$	?
BBB′	0^+	**const**	?	?	**const** $(\boldsymbol{k^4})$	?	?
	1^-	k^2	?	?	k^2 (k^6)	?	?
FFF′	0^+	**const**	$\boldsymbol{a^{-3.332}}^{\blacklozenge}$	?	k^4 (k^8)	?	?
	1^-	k^2	?	?	$\boldsymbol{k^2}$ $(\boldsymbol{k^6})$	$\boldsymbol{a^6}^{\lozenge}$	?
	2^+	k^4	?	?	k^4 (k^8)	?	?

$^{\triangle}$ Fedichev, Reynolds, and Shlyapnikov, PRL **77**, 2921 (1996); Esry, Greene, and Burke, PRL **83**, 1751 (1999).

$^{\blacktriangle}$ D'Incao, Suno, and Esry, PRL **93**, 123201 (2004).

$^{\lozenge}$ Petrov, PRA **67**, 010703(R) (2003).

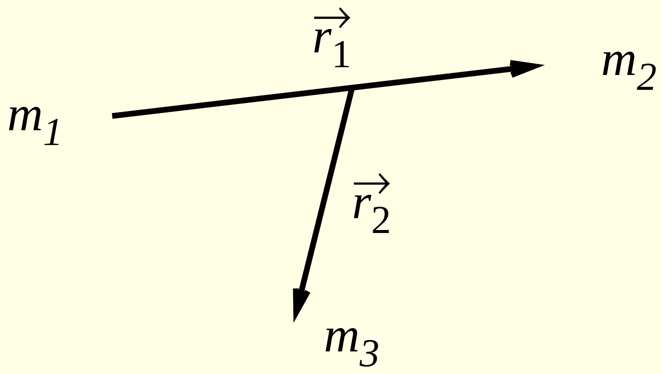
$^{\blacklozenge}$ Petrov, Salomon, and Shlyapnikov, PRL **93**, 090404 (2004).

No general rule for the a scaling laws !

(No simple physcal picture for the know results)

Three-Body Effective Potentials

Hyperspherical Adiabatic Representation



Hyperradius: $R^2 = r_1^2 + r_2^2$ (Overall size)
Hyperangles: Internal motion (Think : Born-Oppenheimer !)

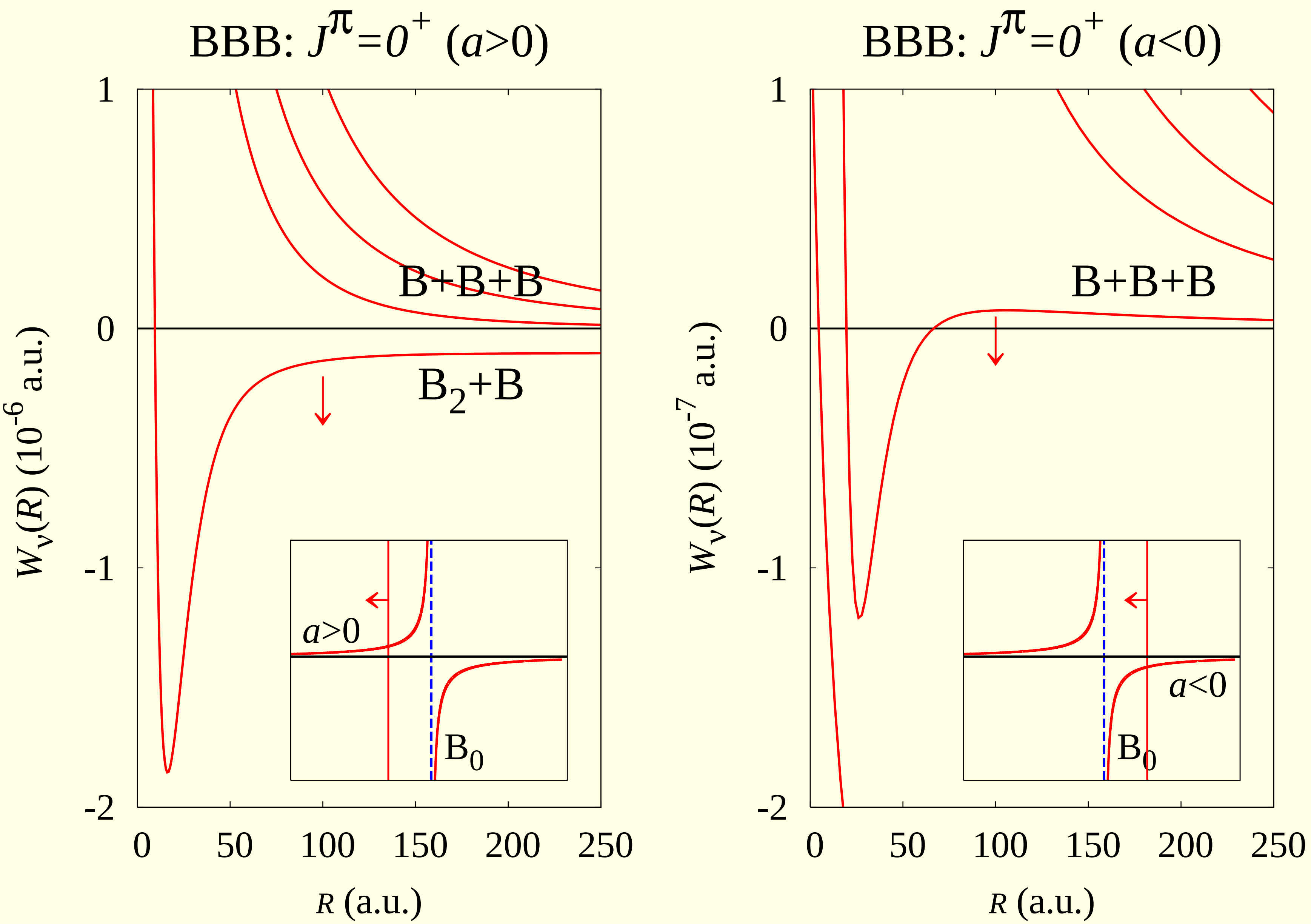
Radial Schrödinger Equation

$$\left[-\frac{1}{2\mu}\frac{d^2}{dR^2} + W_\nu(R)\right] F_\nu(R) + \sum_{\nu'\neq\nu} V_{\nu\nu'}(R) F_{\nu'}(R) = E F_\nu(R)$$

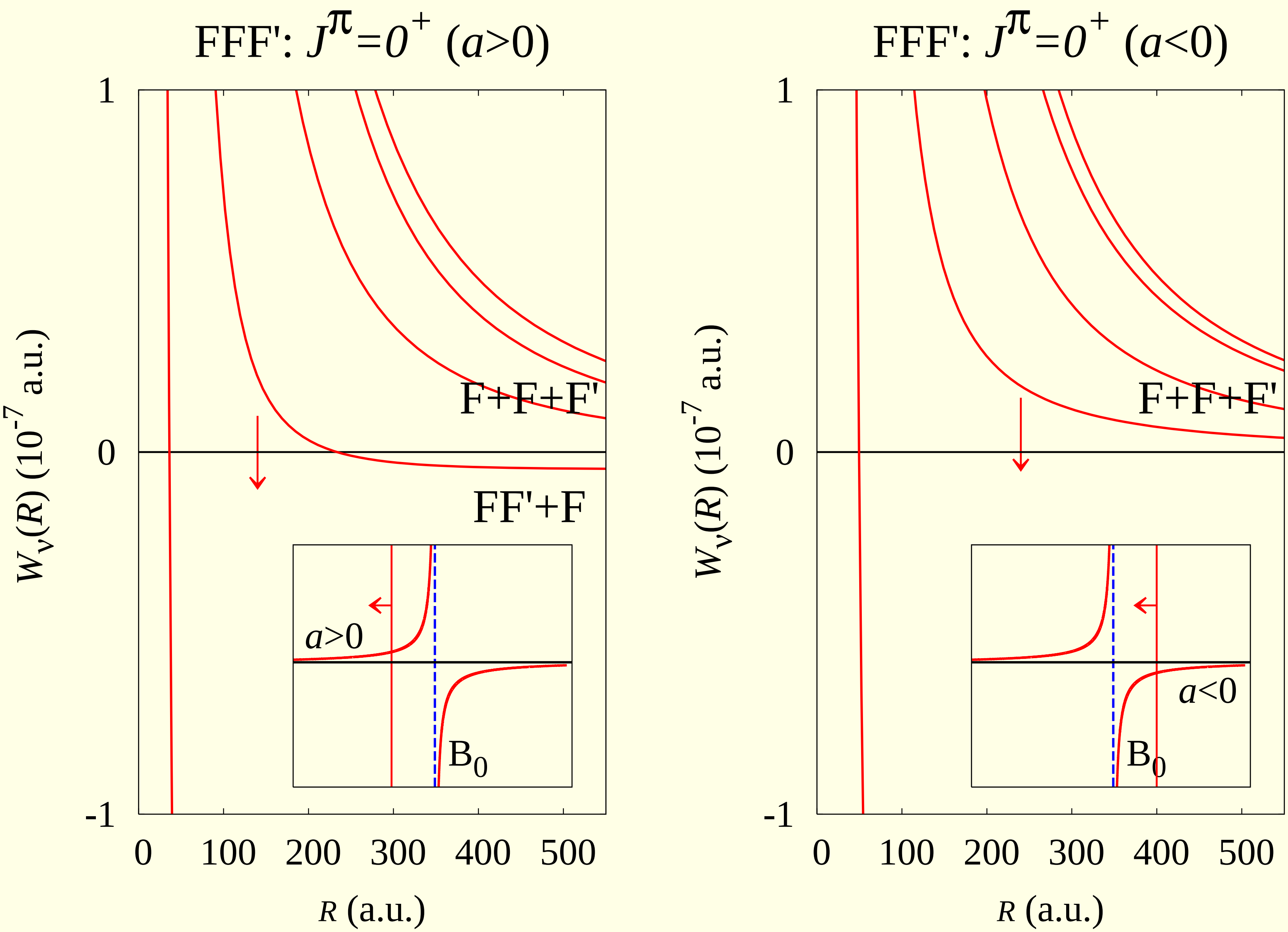
$W_\nu(R) \Rightarrow$ Three-Body Effective potentials

$V_{\nu\nu'}(R) \Rightarrow$ Nonadiabatic Couplings (inelastic transitions)

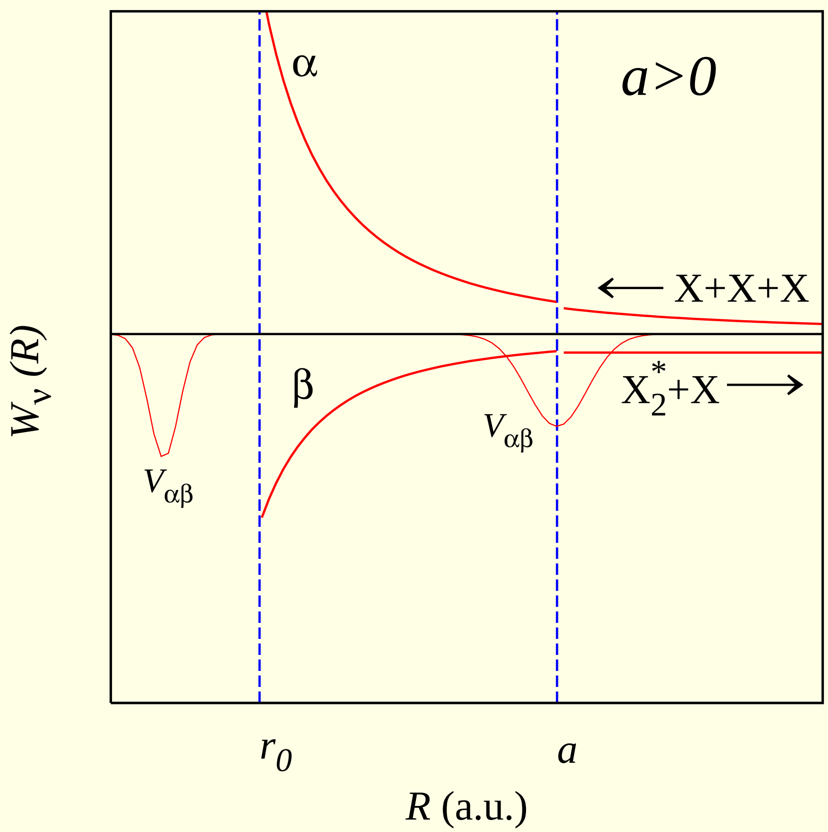
THREE-BODY EFFECTIVE POTENTIALS



Three-Body Effective Potentials



SO, WHAT ABOUT THE INELASTIC TRANSITIONS ?



THREE-BODY RECOMBINATION:

BBB $(J^\pi = 0^+)$

$$X + X + X \to X_2^* + X + E_{vl}$$

$$K_3 \propto a^4 \ !$$

Inelastic Transitions

Tunneling process to where the coupling peaks !

Tunneling Probability (WKB):

$$|T_{fi}|^2 = P^{(\nu)}_{x\to y} \approx \exp\left[-2\int_y^x \sqrt{2\mu\left(W_\nu(R) + \frac{1/4}{2\mu R^2} - E\right)}\, dR\right]$$

x : classical turning point

y : coupling peak position

$$K_3 \propto \frac{|T_{fi}|^2}{k^4} \qquad V_{\text{rel}} \propto \frac{|T_{fi}|^2}{k}$$

Three-Body Effective Potentials

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If $W_\nu(R)$ is different for each 3-body system ?

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If $W_\nu(R)$ is different for each 3-body system ?

If $V_{\nu\nu'}(R)$ peaks in any place ?

... But ... That's not the case !

Three-Body Effective Potentials

3-Body systems (s-wave res. int.

Two Categories !

For $\|a\| \gg r_0$		$R < r_0$ (Not Universal)	$r_0 < R < \|a\|$ (Universal)	$R > \|a\|$ (Universal)
Category I	$\rightarrow$	Details	$-\frac{s_0^2+1/4}{2\mu R^2}$ or $\frac{s_\nu^2-1/4}{2\mu R^2}$	$E_{vl'}+\frac{l(l+1)}{2\mu R^2}$ or $\frac{\lambda(\lambda+4)+15/4}{2\mu R^2}$
Category II	$\rightarrow$	Details	$\frac{p_0^2+1/4}{2\mu R^2}$ or $\frac{p_\nu^2-1/4}{2\mu R^2}$	$E_{vl'}+\frac{l(l+1)}{2\mu R^2}$ or $\frac{\lambda(\lambda+4)+15/4}{2\mu R^2}$

l, l' - Angular Momentum $\rightarrow$ Symmetry Dep.

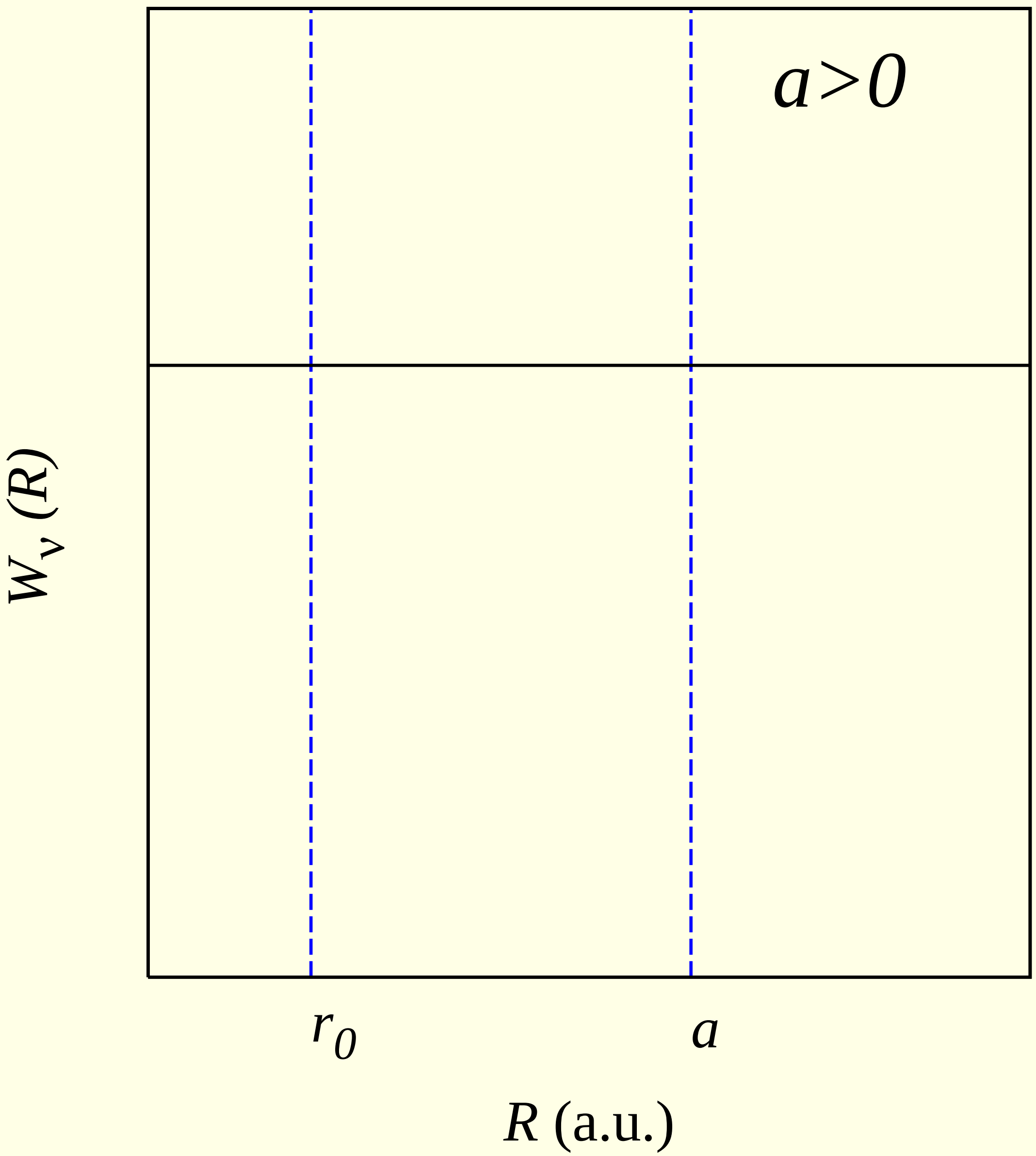
λ - Kinetic Energy $\rightarrow$ Symmetry Dep.

s_0, s_ν, p_0 and p_ν - Efimov Physics $\rightarrow$ Symmetry Dep. !

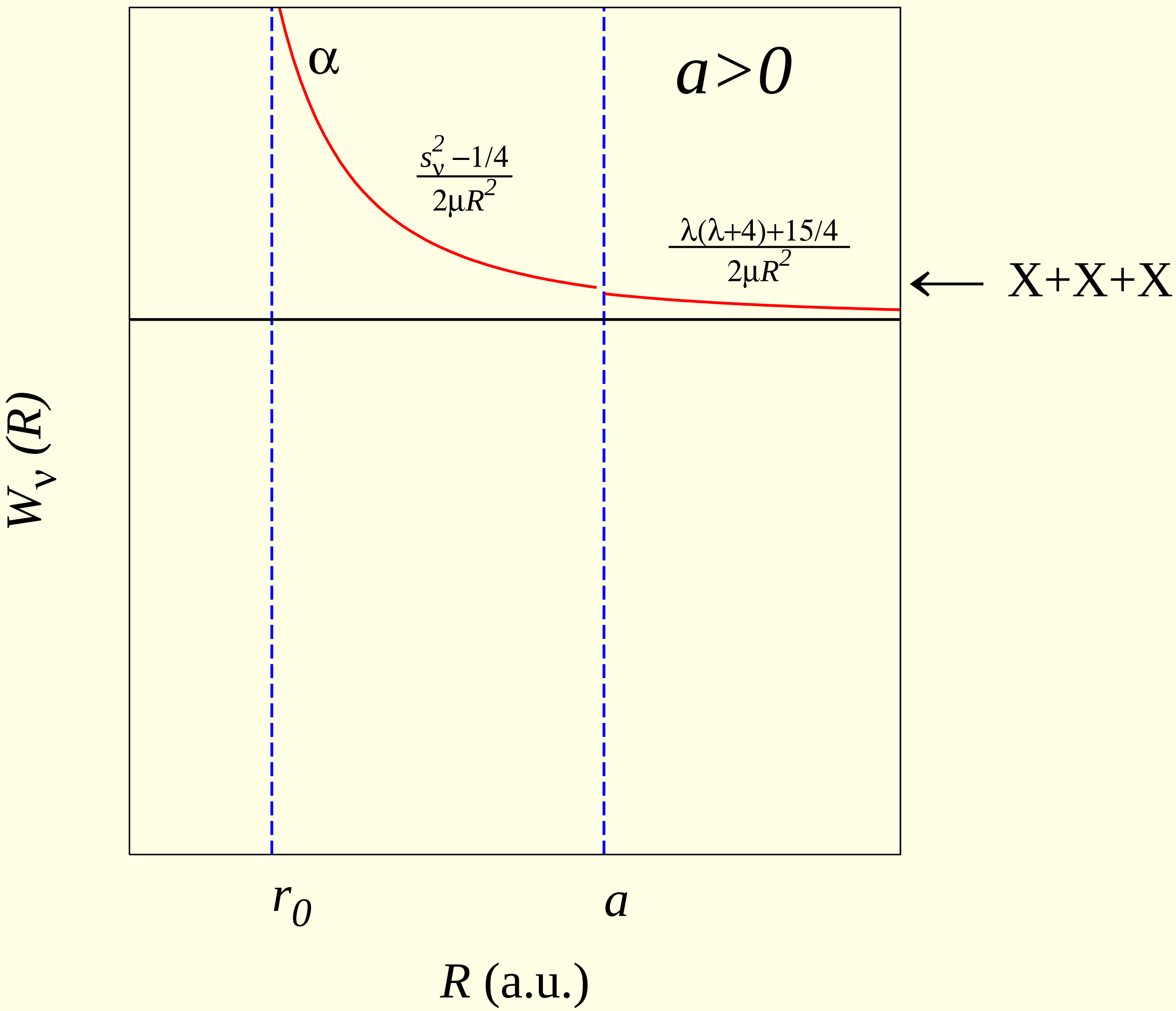
$\rightarrow$ Number of Resonant Interactions !

$\rightarrow$ Mass Ratios !

Three-Body Effective Potentials



Three-Body Effective Potentials



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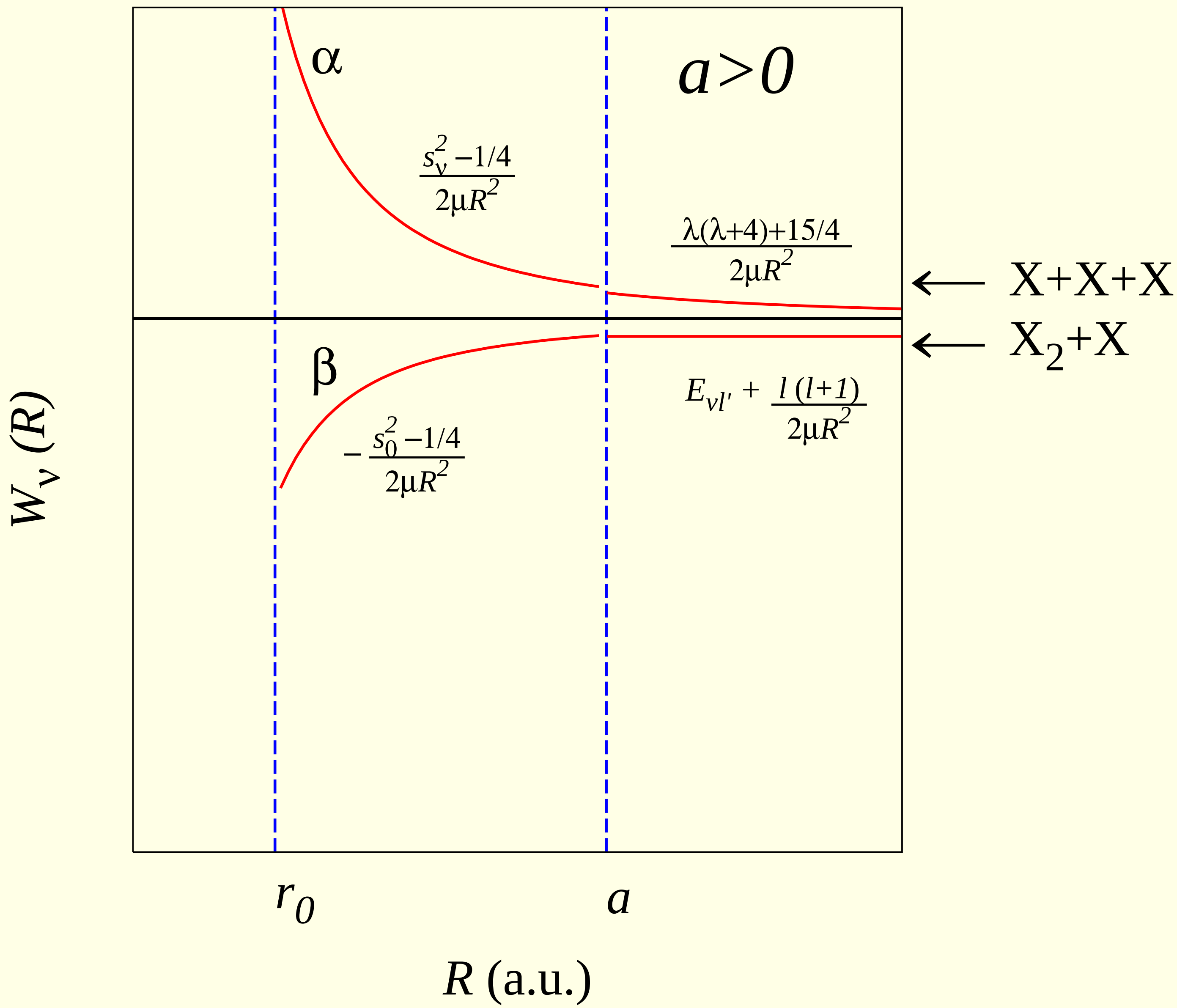

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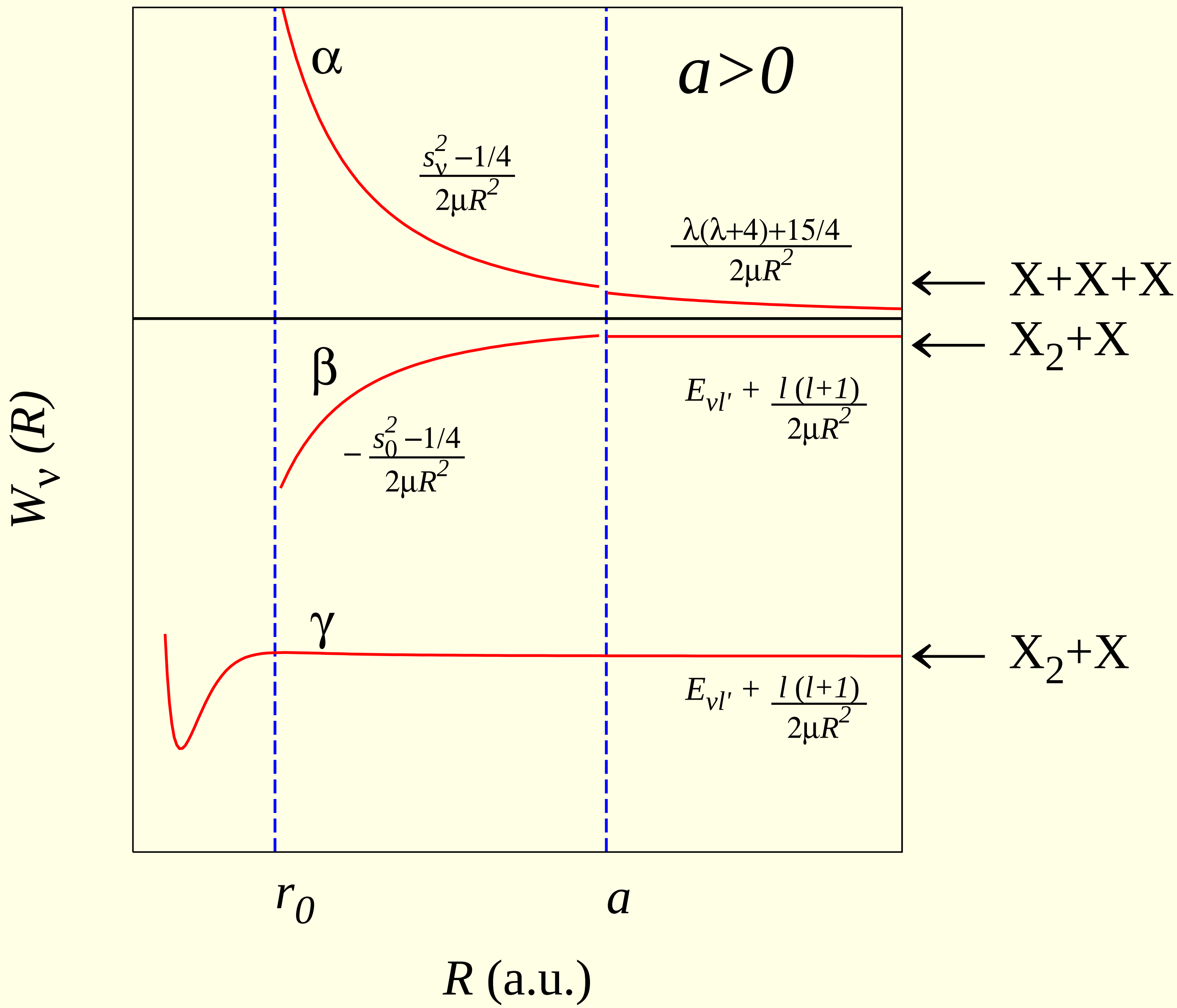

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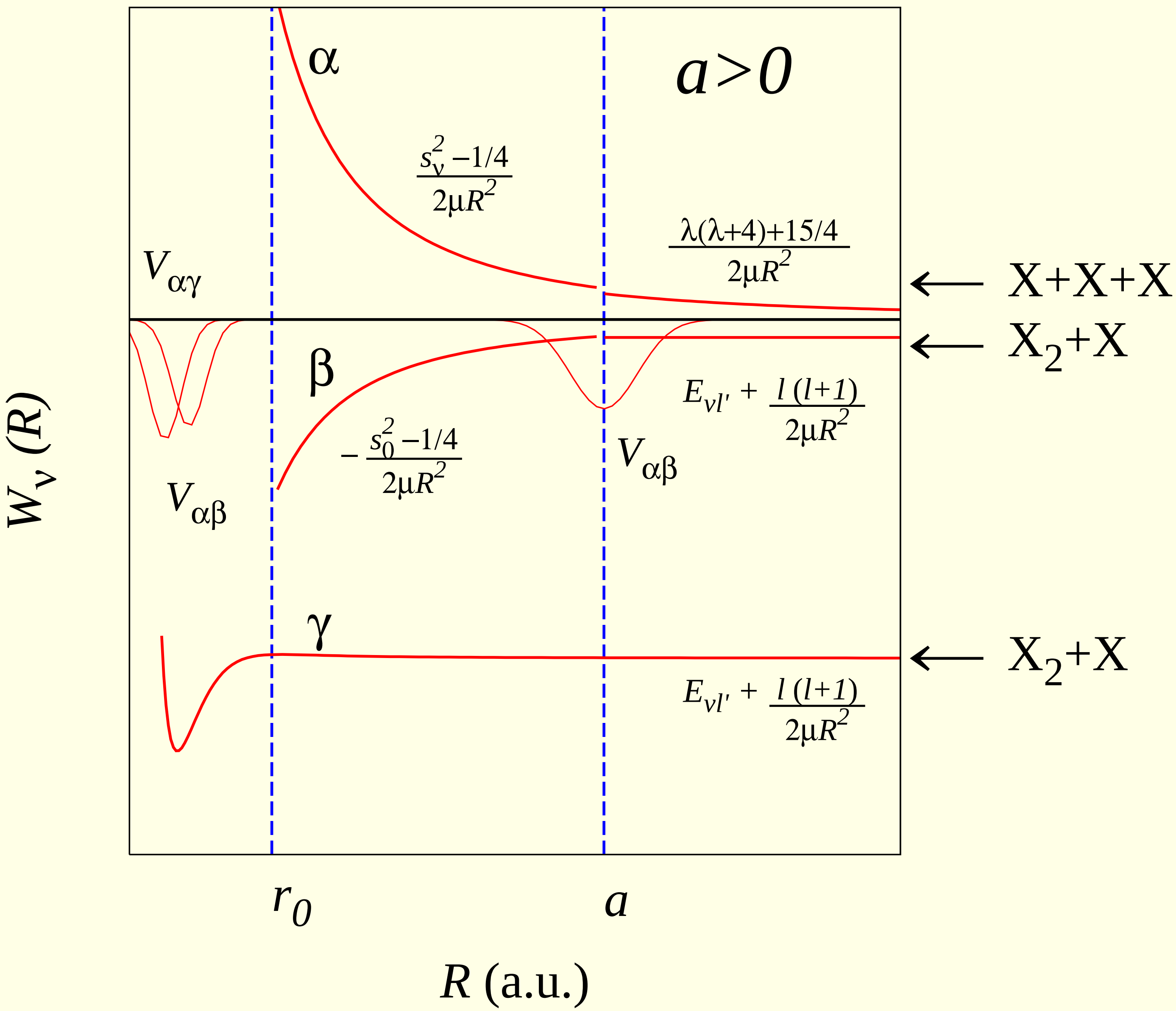

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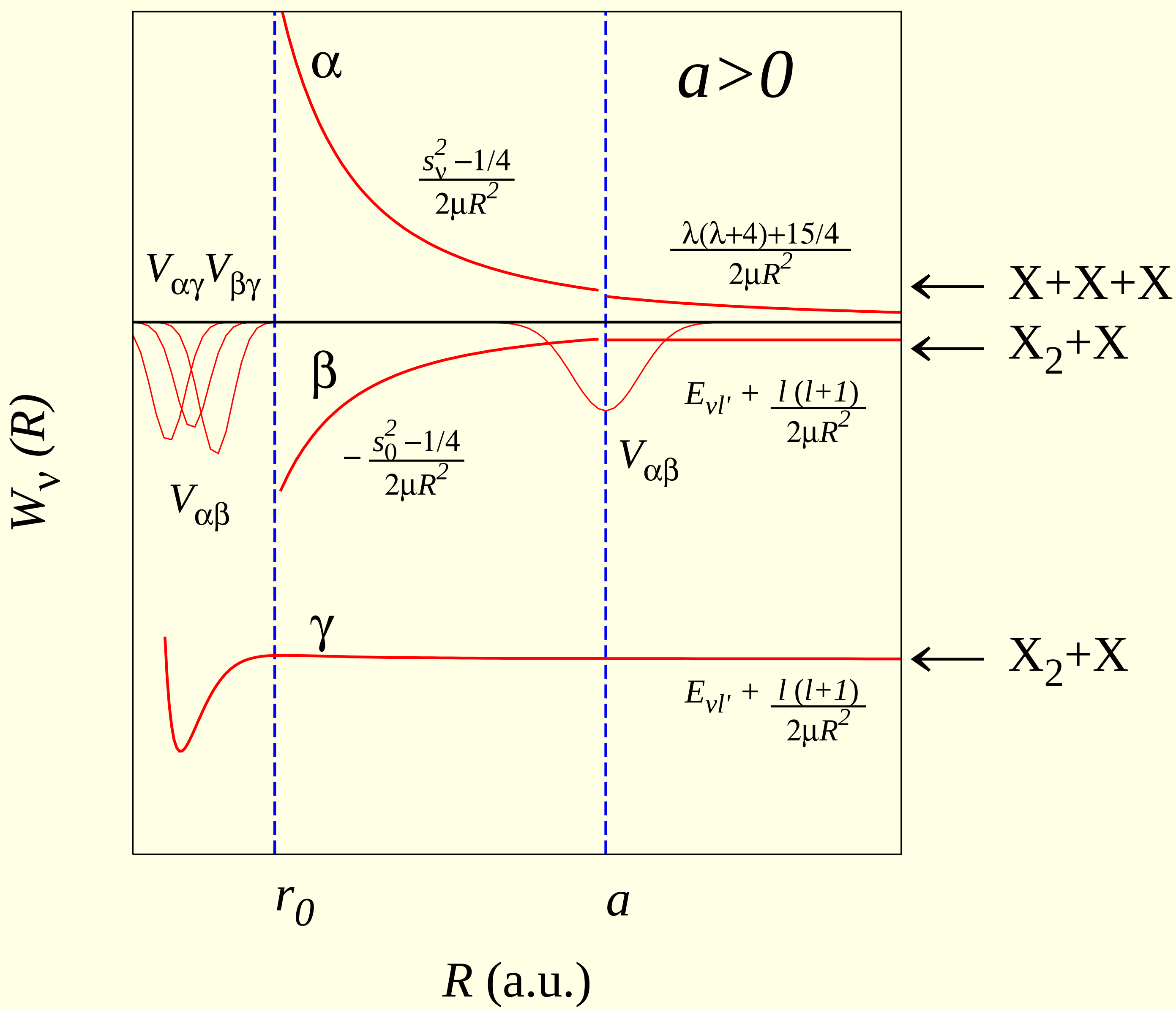
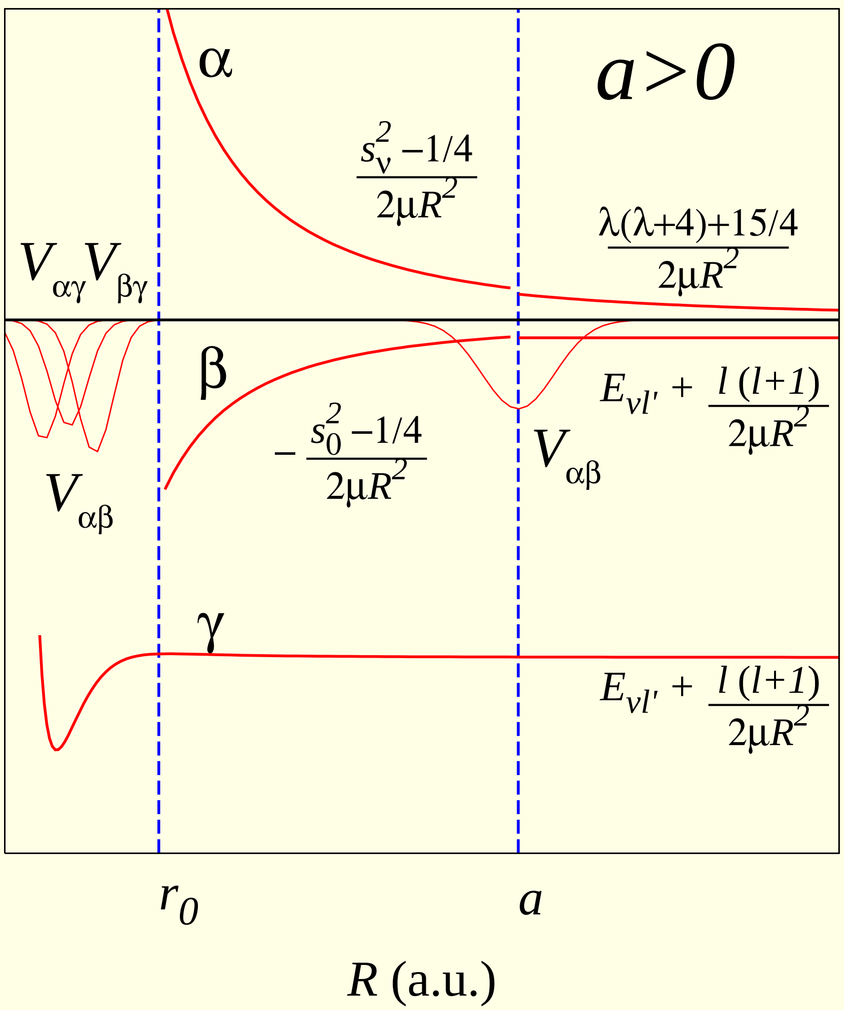
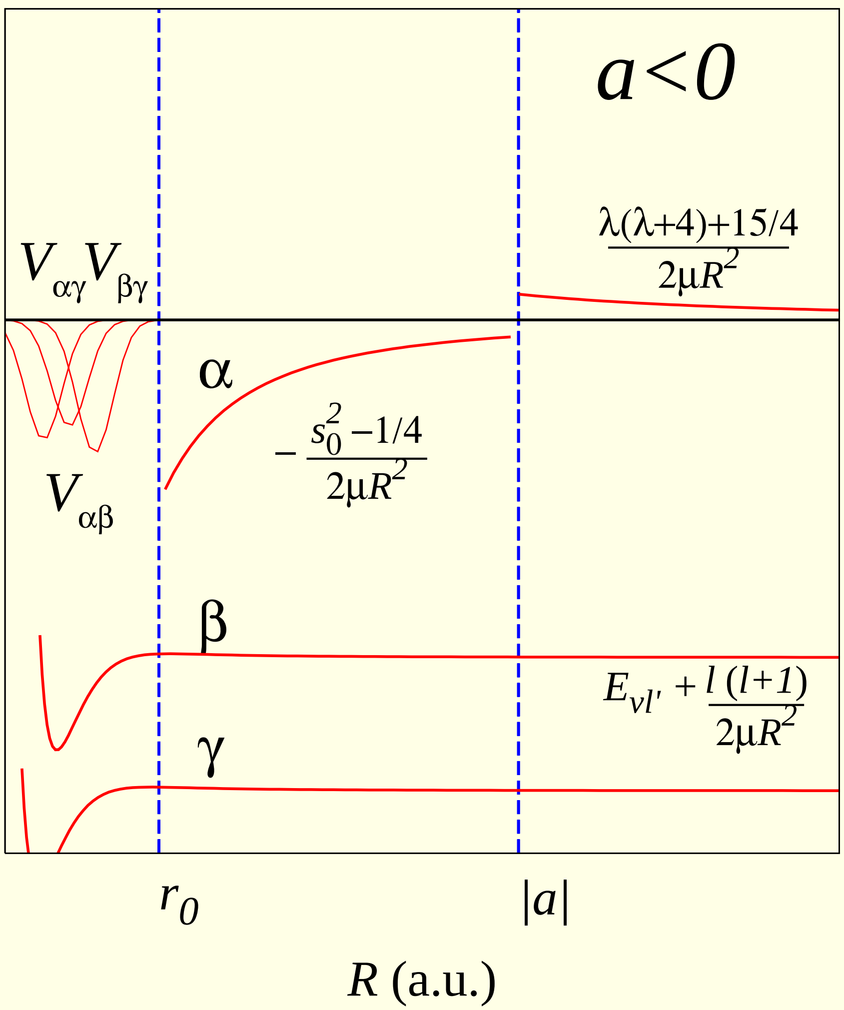


Three-Body Effective Potentials

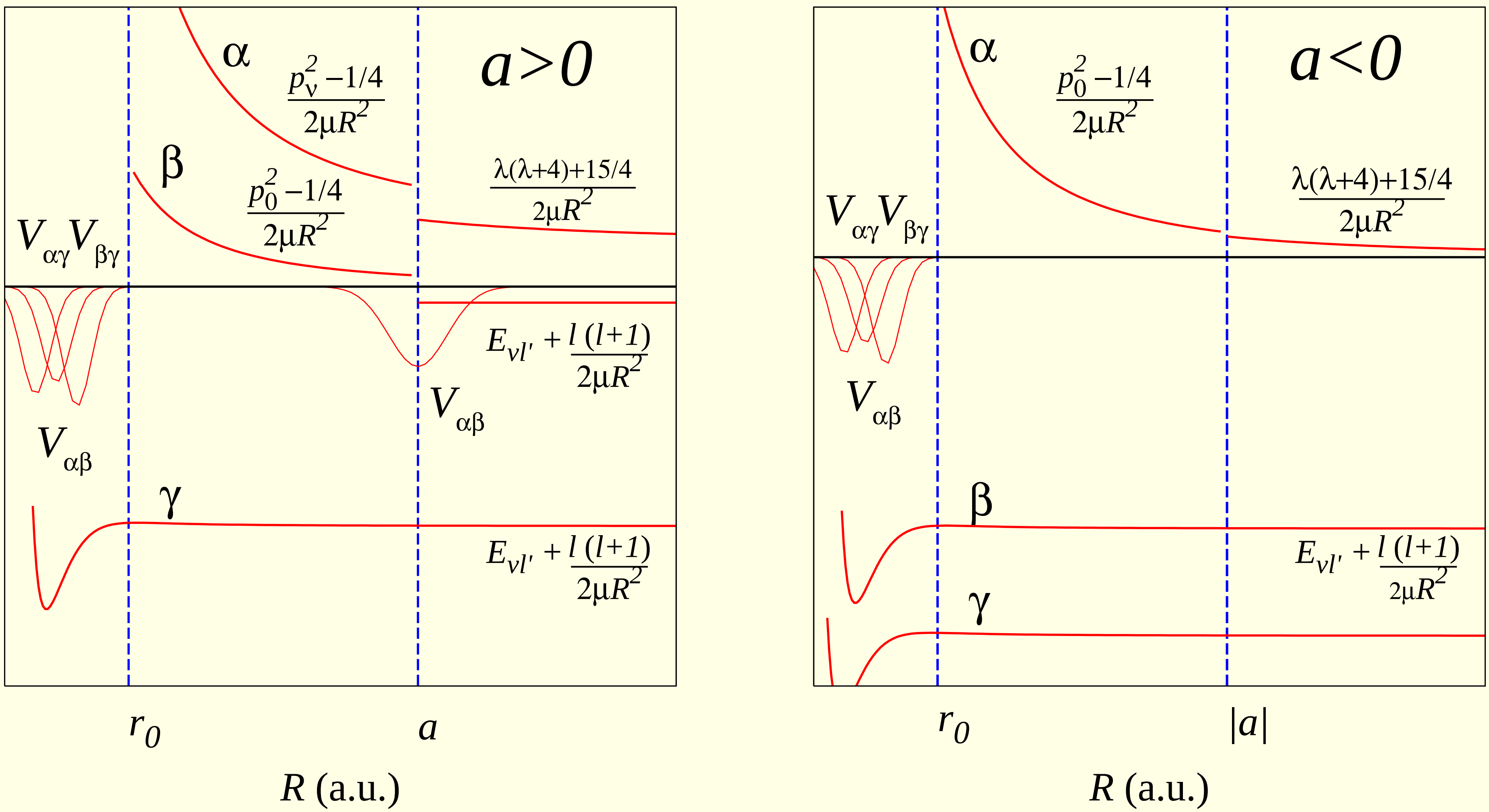
Category I





Three-Body Effective Potentials

Category II



Three-Body Effective Potentials

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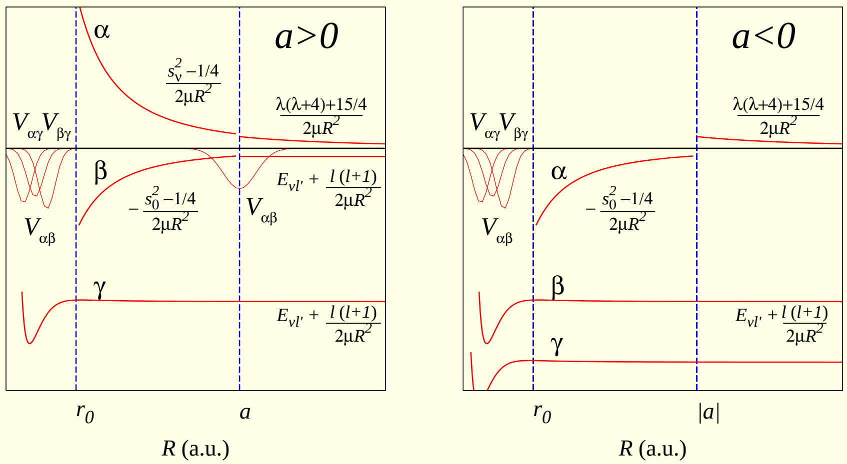

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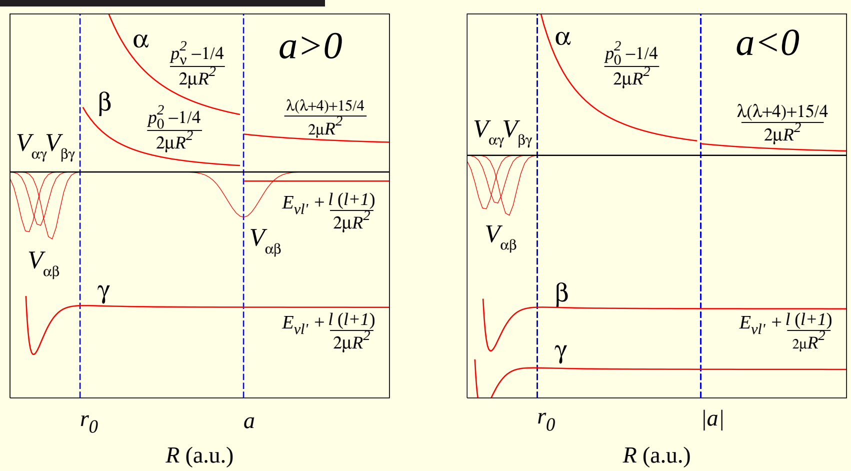


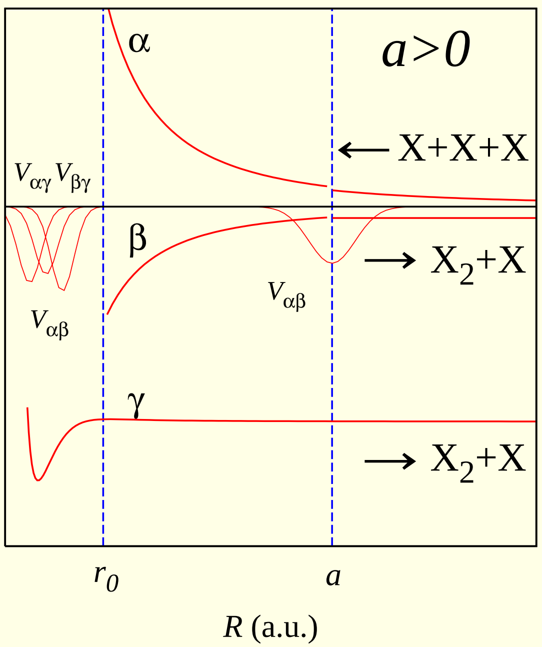
+ WKB ⇒ **3-Body Rates**

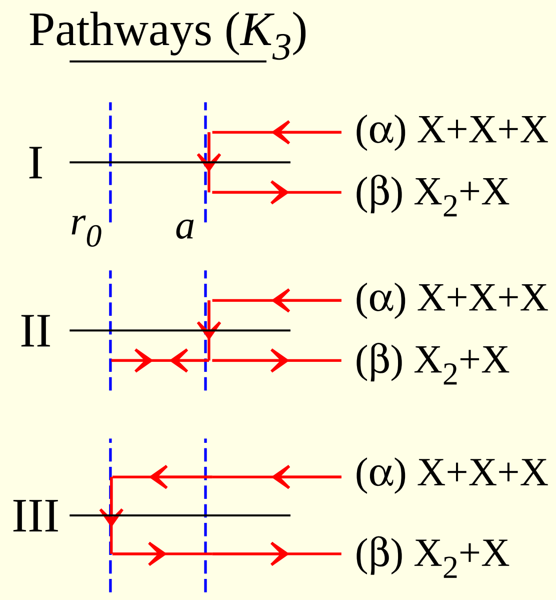
Threshold Laws

***a*-Scaling Laws**

Three-Body Recombination K_3

Category I

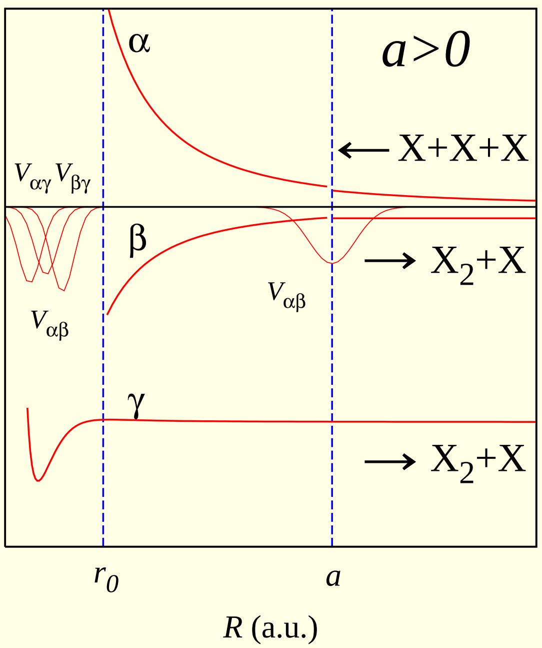




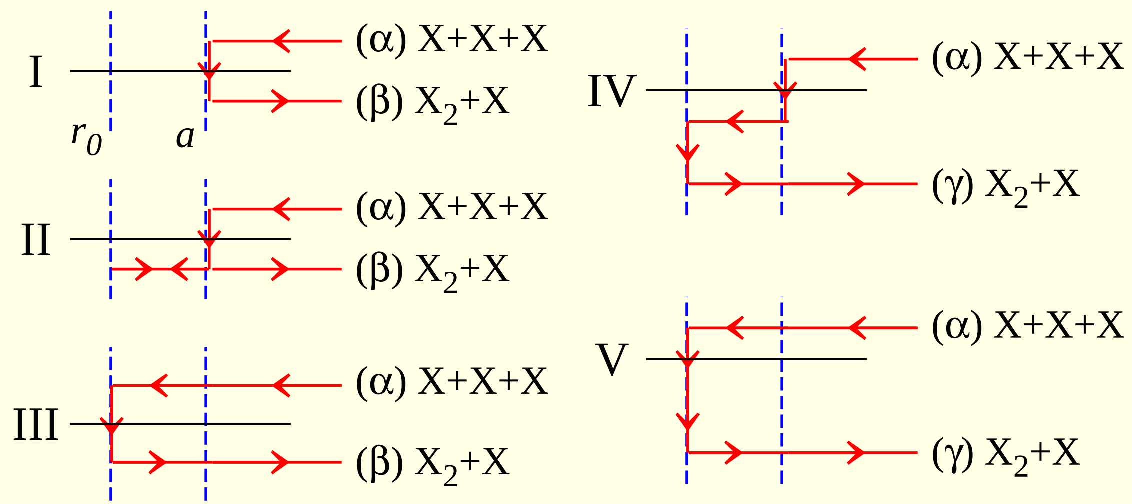
$$K_3 \propto k^{2\lambda}\left[A_\eta \sin^2\left[s_0 \ln(a/r_0) + \Phi\right] a^{2\lambda+4} + B_\eta \left(\frac{r_0}{a}\right)^{2s_\nu} a^{2\lambda+4}\right] + \ldots$$

Three-Body Recombination K_3

Category I



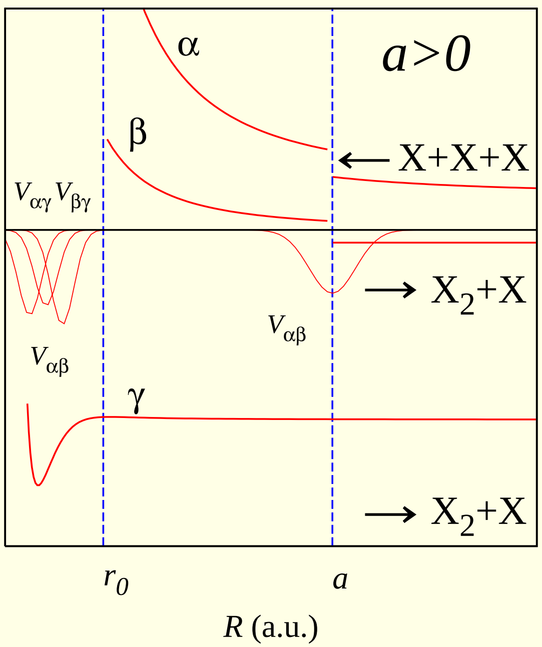
Pathways (K_3)



$$K_3 \propto k^{2\lambda}\left[A_\eta \sin^2\left[s_0 \ln(a/r_0)+\Phi\right] a^{2\lambda+4} + B_\eta \left(\frac{r_0}{a}\right)^{2s_\nu} a^{2\lambda+4}\right] + \left[C_\eta \left(\frac{r_0}{a}\right)^{2s_\nu} a^{2\lambda+4} + D_\eta a^{2\lambda+4}\right]$$

Three-Body Recombination K_3

Category II



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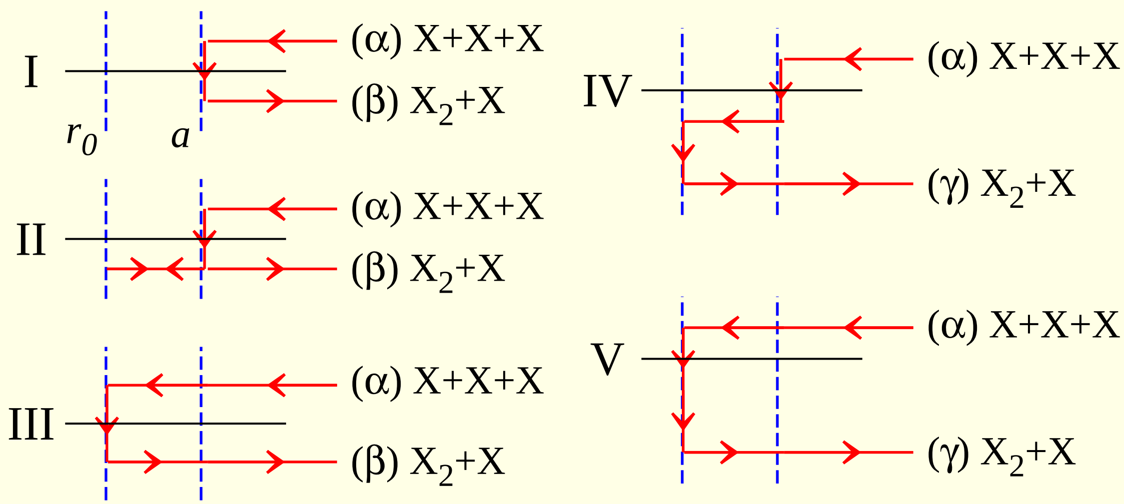


$$K_3 \propto k^{2\lambda}\left[A_\eta + B_\eta\left(\frac{r_0}{a}\right)^{2p_0} + C_\eta\left(\frac{r_0}{a}\right)^{2p_\nu} + D_\eta\left(\frac{r_0}{a}\right)^{2p_0+2p_\nu}\right] a^{2\lambda+4}$$

Three-Body Rates

Category I

- $(a > 0)$ $K_3 \propto k^{2\lambda}\left[A_\eta \sin^2\left[s_0 \ln(a/r_0) + \Phi\right] + B_\eta \left(\frac{r_0}{a}\right)^{2s_\nu} + C_\eta\right] a^{2\lambda+4}$
- $(a < 0)$ $K_3 \propto k^{2\lambda}\left[\frac{\sinh(2\eta)}{\sin^2\left[s_0 \ln(|a|/r_0) + \Phi\right] + \sinh^2(\eta)}\right] a^{2\lambda+4}$

- $(a > 0)$ $V_{\rm rel} \propto k^{2l}\left[\frac{\sinh(2\eta)}{\sin^2\left[s_0 \ln(|a|/r_0) + \Phi\right] + \sinh^2(\eta)}\right] a^{2l+1}$
- $(a < 0)$ $V_{\rm rel} \propto A_\eta k^{2l} r_0^{2l+1}$

Includes: BBB $(J^\pi = 0^+)$, BBB′ $(J^\pi = 0^+)$, BBb $(J^\pi = 0^+)$,
FFf $(J^\pi = 1^-,\ m_{\rm f}/m_{\rm F} < 0.073)$, ...

Three-Body Rates

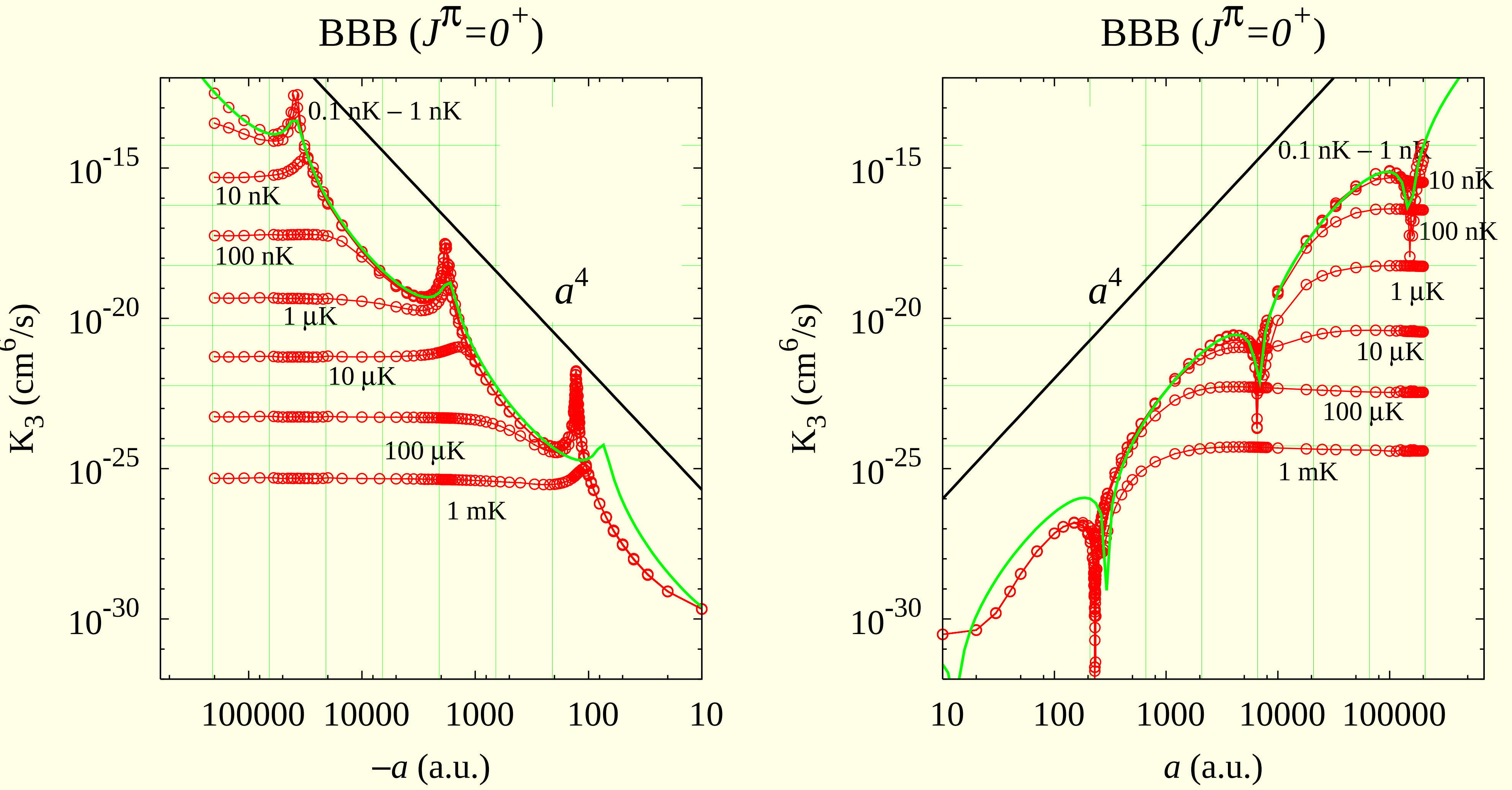
Category II

- $(a > 0)$ $\quad K_3 \propto k^{2\lambda} \left[A_\eta + B_\eta \left(\frac{r_0}{a}\right)^{2p_0} + C_\eta \left(\frac{r_0}{a}\right)^{2p_\nu} + D_\eta \left(\frac{r_0}{a}\right)^{2p_0+2p_\nu} \right] a^{2\lambda+4}$
- $(a < 0)$ $\quad K_3 \propto k^{2\lambda} \left(\frac{r_0}{|a|}\right)^{2p_0} |a|^{2\lambda+4}$

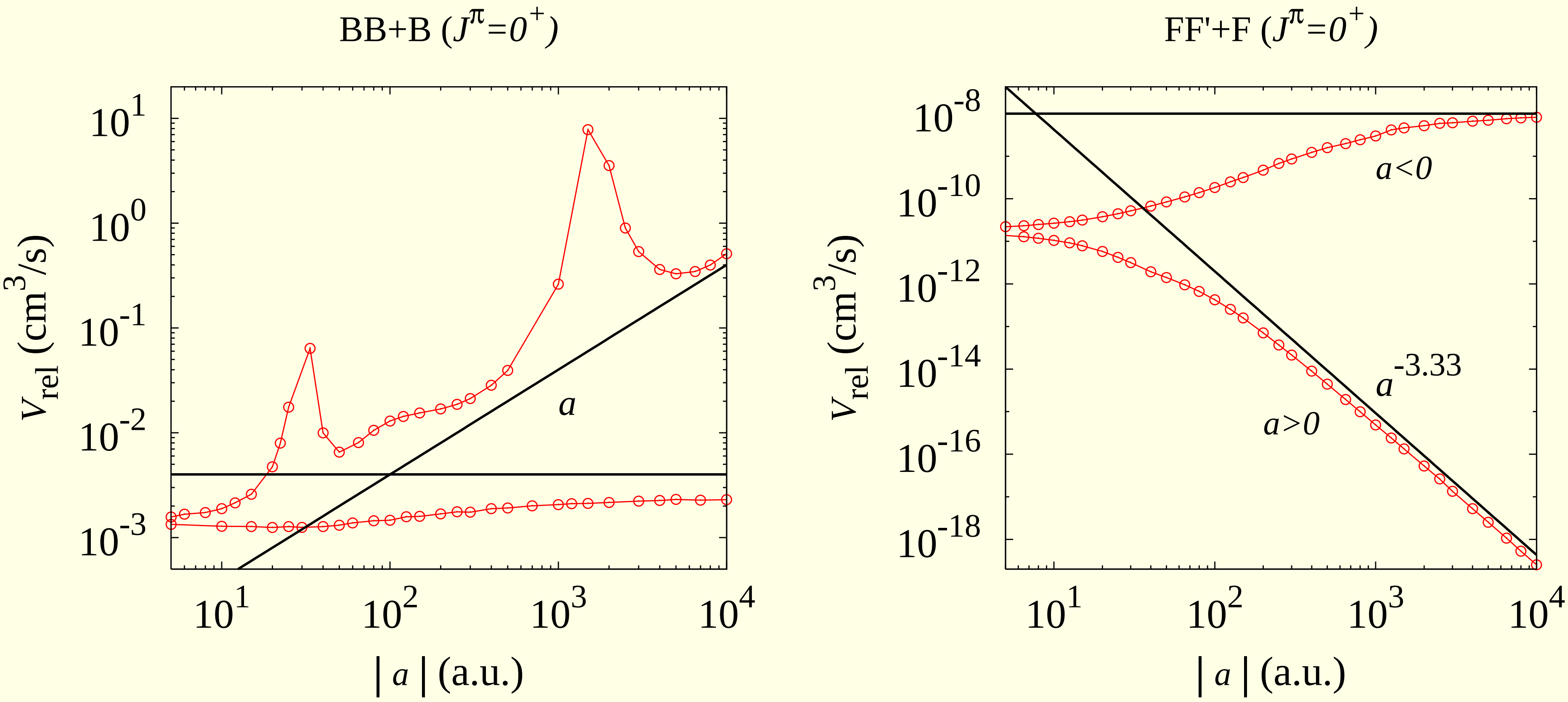
- $(a > 0)$ $\quad V_{\rm rel} \propto A_\eta k^{2l} \left(\frac{r_0}{a}\right)^{2p_0} a^{2l+1}$
- $(a < 0)$ $\quad V_{\rm rel} \propto A_\eta k^{2l} r_0^{2l+1}$

Includes: BBB (J^π=2^+), BBB′ (J^π=2^+),
FFF′ (J^π=0^+), FFf (J^π=0^+), FFf (J^π=1^-, $m_{\rm f}/m_{\rm F} > 0.073$), …

Three-Body Recombination K_3



Vibrational Relaxation V_{rel}



Scaling Laws (Equal Mass)

		V_{rel}			K_3 (D_3)		
	J^π	E	$a > 0$	$a < 0$	E	$a > 0$	$a < 0$
BBB	0^+	CONST	a	CONST	CONST (k^4)	a^4	$\|a\|^4$
	1^-	k^2	$a^{-2.728}$	CONST	k^6 (k^{10})	a^{10}	$\|a\|^{4.272}$
	2^+	k^4	$a^{-0.647}$	CONST	k^4 (k^8)	a^8	$\|a\|^{2.353}$
BBB′	0^+	CONST	a	CONST	CONST (k^4)	a^4	$\|a\|^4$
	1^-	k^2	$a^{-1.558}$	CONST	k^2 (k^6)	a^6	$\|a\|^{1.443}$
FFF′	0^+	CONST	$a^{-3.332}$	CONST	k^4 (k^8)	a^8	$\|a\|^{3.668}$
	1^-	k^2	$a^{-0.546}$	CONST	k^2 (k^6)	a^6	$\|a\|^{2.455}$
	2^+	k^4	$a^{-1.210}$	CONST	k^4 (k^8)	a^8	$\|a\|^{1.790}$

Summary

- Our method describe ALL relevent tree-body systems in terms of simple physical concepts (Tuneling) and fundamental physics (Efimov)

- Both Threshold and a-Scaling Laws depends only in the initial state

- Pervasive influence of the Efimov physics

- Control of the collisional aspects by choosing the masses