Limits on Universality in Ultracold Three-Bosons Recombination

José P. D'Incao, Hiroya Suno and Brett D. Esry

Department of Physics, Kansas State University,

Manhattan, Kansas 66506

Motivations

Ultracold Atomic Gases : Feshbach Resonance

- Scattering Length a : Atomic Interactions

(Nonlinear Excitations, Atomic Solitions, Collapse/Explosion, BEC/BCS)

Main Results : (Bosons)

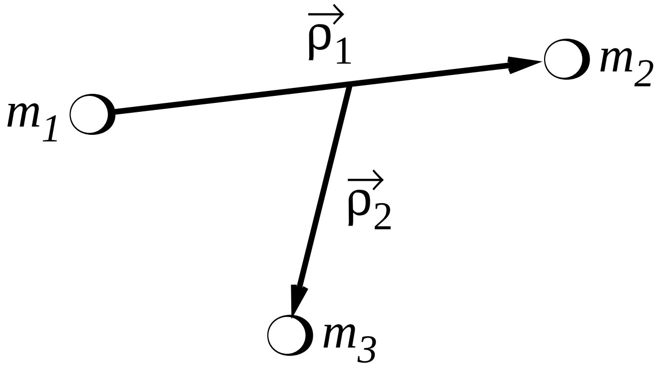
- Three-body recombination : $X + X + X \to X_2 + X + E_{vl}$
- $|a| < 2\,10^5$ a.u., $R \approx 10^7$ a.u., $0.1\ n\text{K} < E < 1\ m\text{K}$
- Universal Behavior for 3-body ultracold collisions

Outline

- Hyperspherical Adiabatic Approach
  - Potential Model
  - $a > 0$: Interference Mechanism
  - $a < 0$: Resonance Mechanism

- Universal behavior
  - Finite Energy Calculations
  - Restrictions on the universal behavior

Hyperspherical Adiabatic Approach



Mass-scaled Jacobi coordinates

$$\vec{\rho}_1 = (\vec{r}_1 - \vec{r}_2)/d$$

$$\vec{\rho}_2 = d\,[\vec{r}_3 - (m_1\vec{r}_1 + m_2\vec{r}_2)/(m_1 + m_2)]$$

Spherical Coordinates $\Rightarrow$ Hyperspherical Coordinates

Spherical Coordinates	Hyperspherical Coordinates
$\rho_1,\ \rho_2\ :\ [0, +\infty)$	$R^2 = \rho_1^2 + \rho_2^2\ :\ [0, +\infty)$
(+ 4-angles)	(+ 5-angles, Ω)

Schrödinger Equation

$$-\frac{1}{2\mu}\left(\frac{d^2}{d^2R}+U_\nu(R)\right)F_\nu(R)$$
$$-\sum_{\nu'}\left(2P_{\nu\nu'}(R)\frac{d}{dR}+Q_{\nu\nu'}(R)\right)F_{\nu'}(R)=EF_\nu(R)$$

$U_\nu(R) \Rightarrow$ Particles Interactions

$Q_{\nu\nu'}(R), P_{\nu\nu'}(R) \Rightarrow$ Inelastic transitions

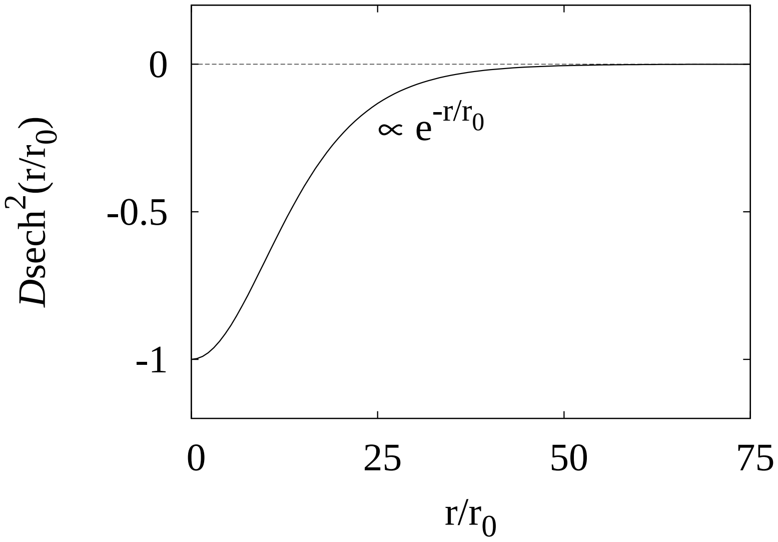
Three-Body Recombination Rate

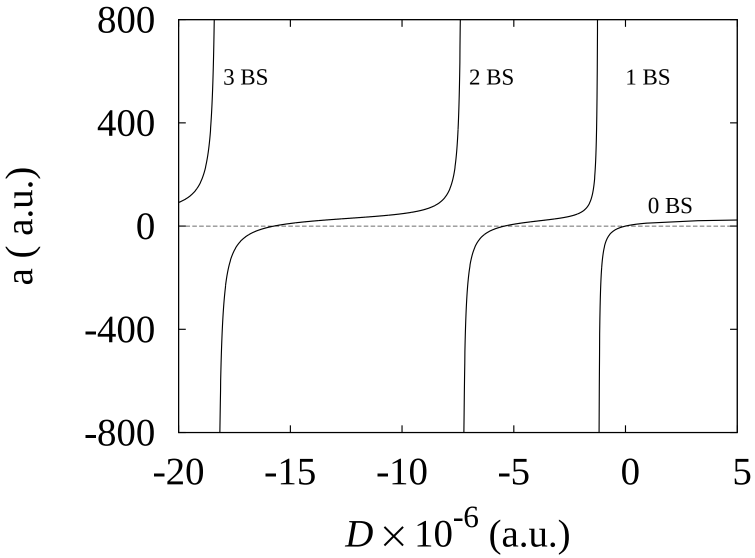
$$K_3=\frac{\hbar k}{\mu}\sigma=\sum_{J,\pi}\sum_{i,f}\frac{192(2J+1)\pi^2}{\mu k^4}|S^{J\pi}_{f\leftarrow i}|^2, \quad k=\sqrt{2\mu E}$$

Potential Model

$$v(r_{ij}) = D\text{sech}^2(r_{ij}/r_0) \qquad D \equiv \text{potential depth}$$

$$r_0 \equiv \text{characteristic range}$$



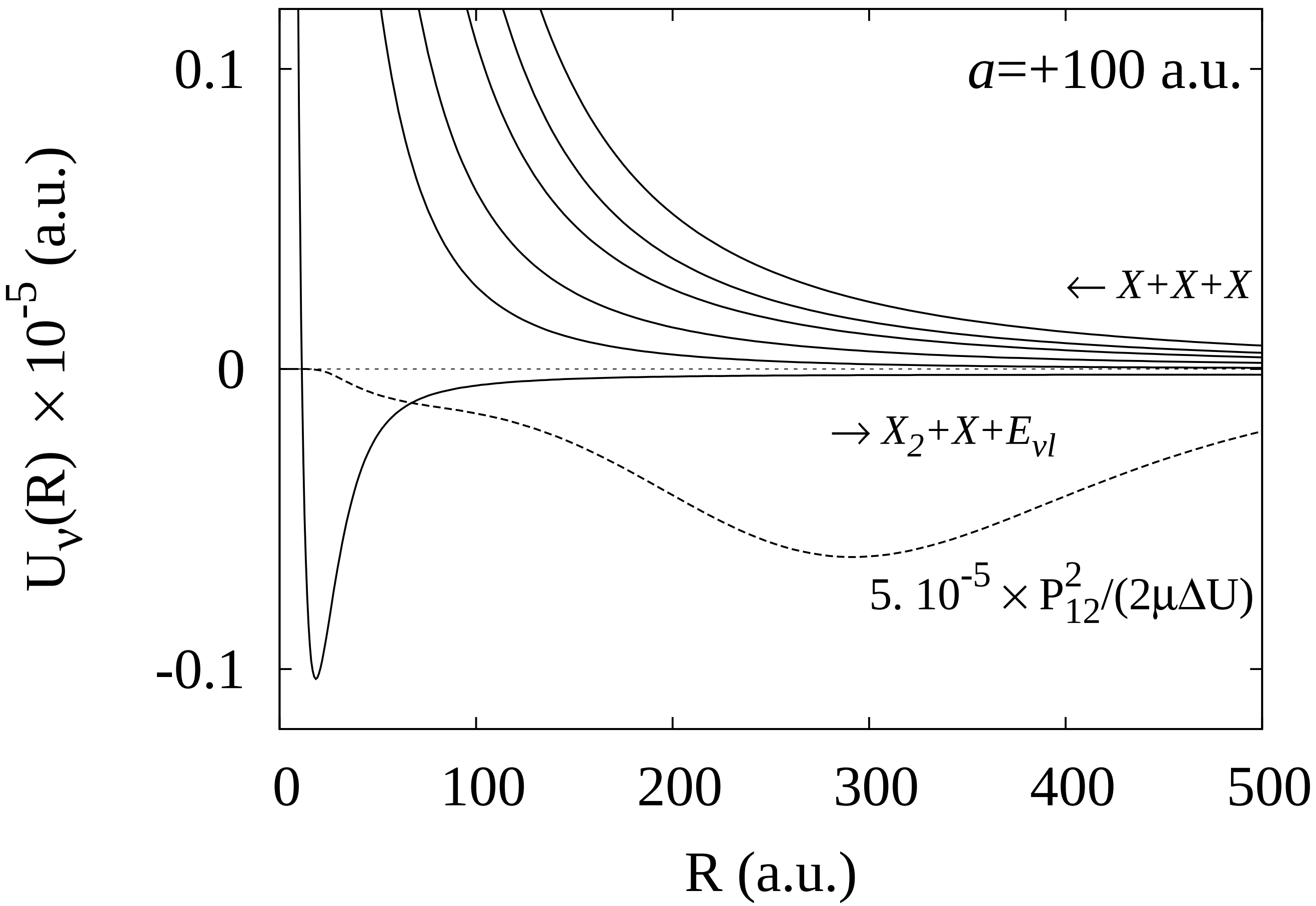


- $a > 0$: repulsive interactions
- $a < 0$: attractive interactions

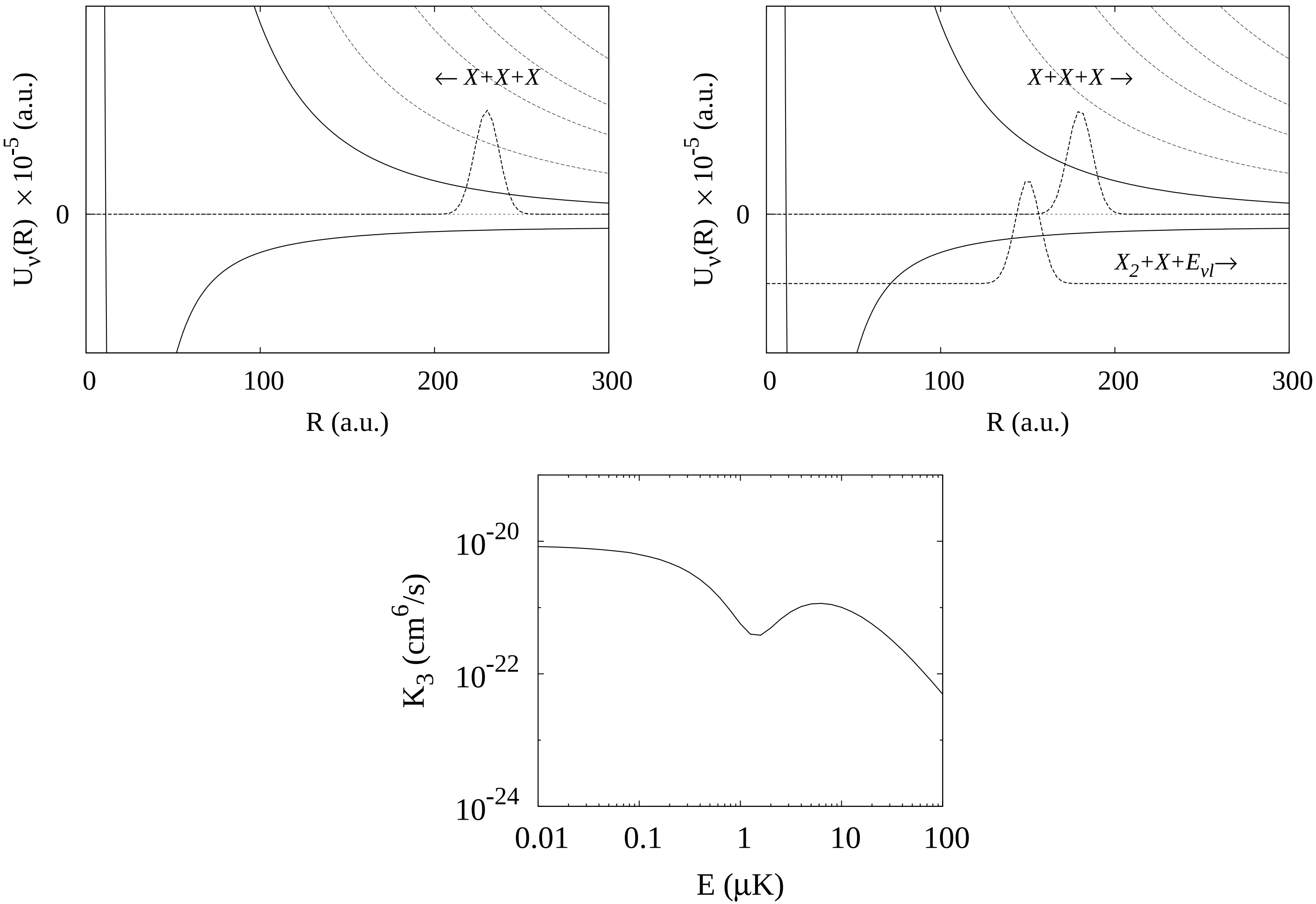
$$V_{int}(R, \Omega) = v(r_{12}) + v(r_{13}) + v(r_{23})$$

Three-Bosons System with $a > 0$

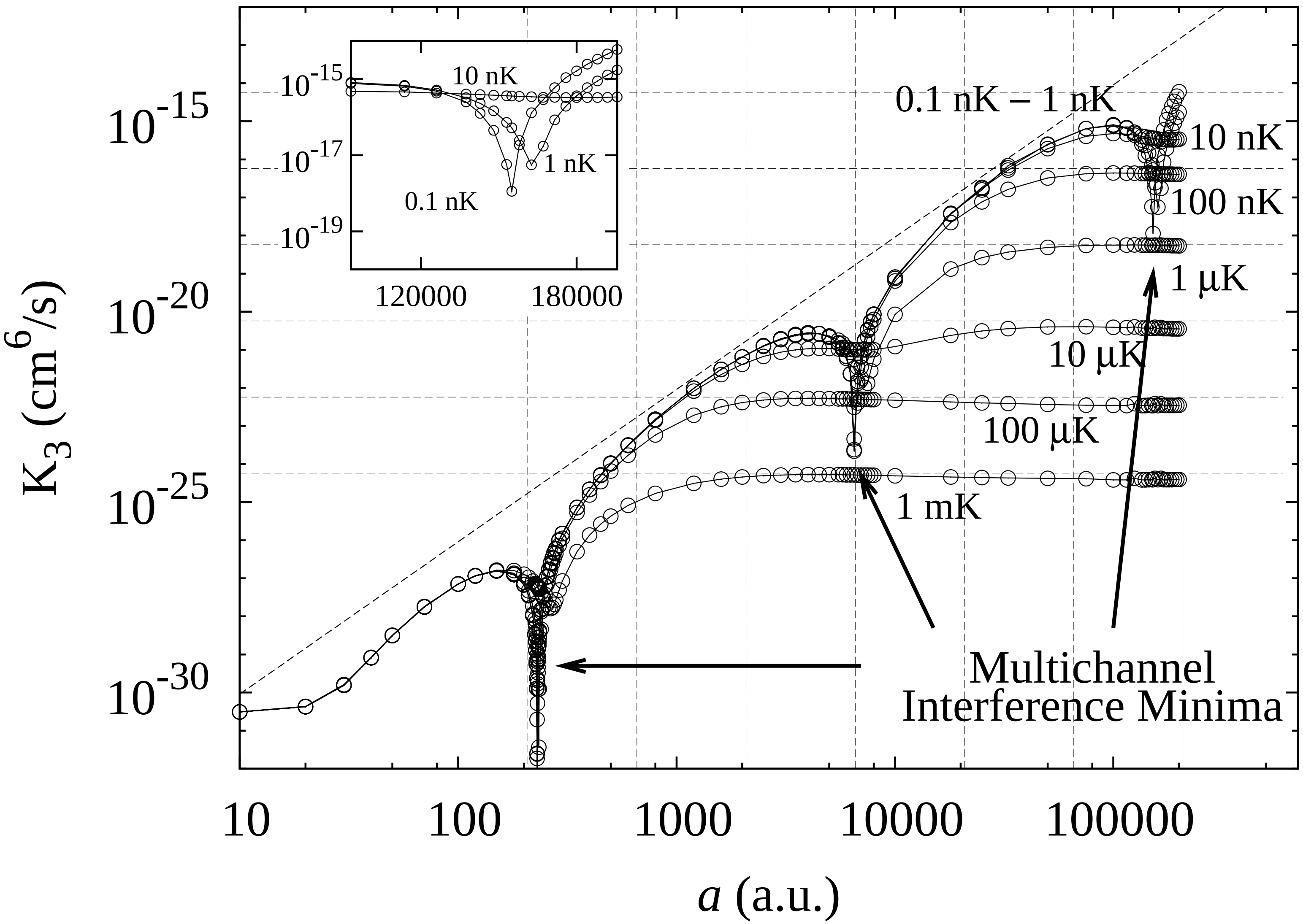
(Broad Coupling at $R \sim 3a$)



Recombination Rate for $a > 0$ (Interference)



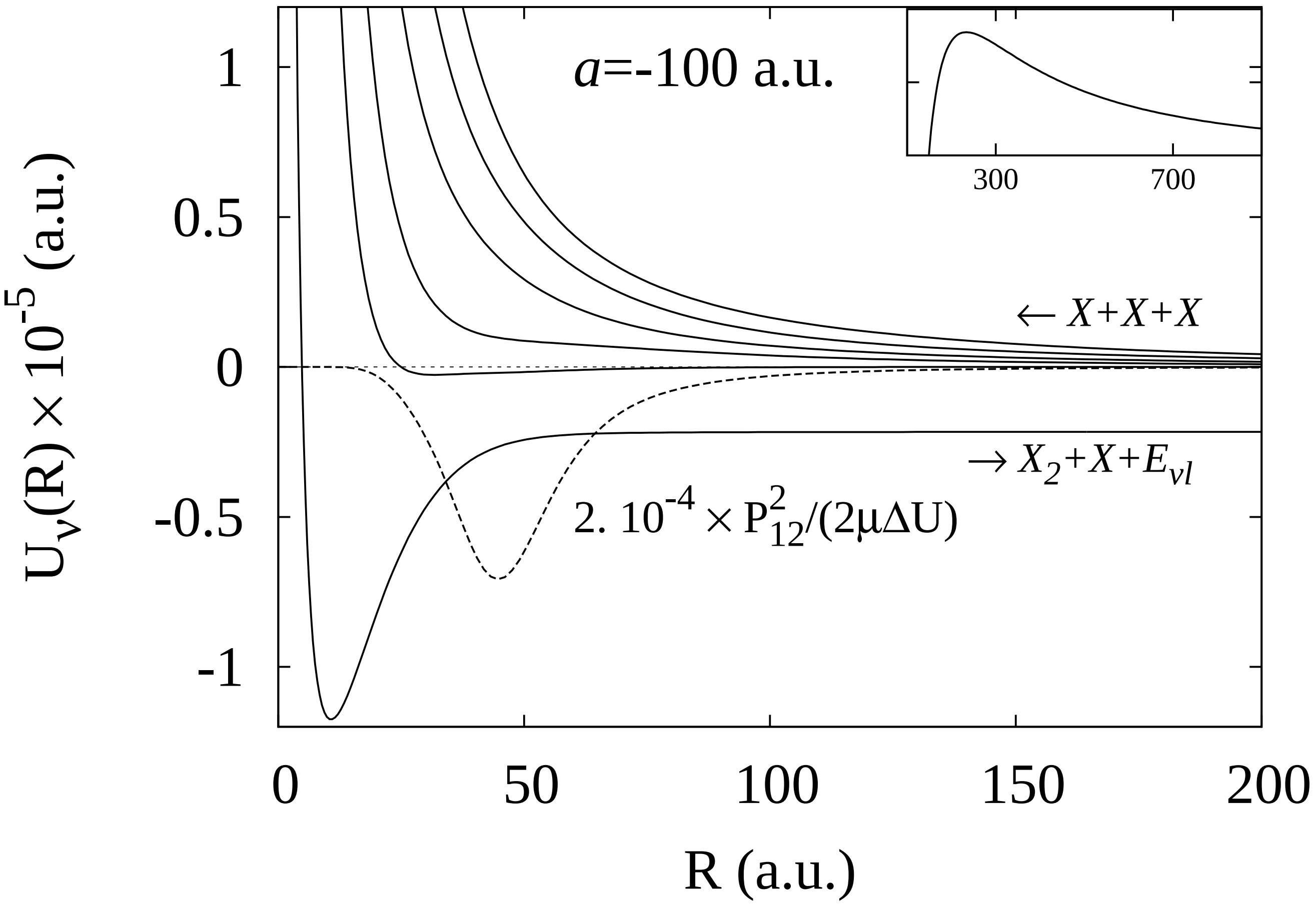
General Feature ($a > 0$)



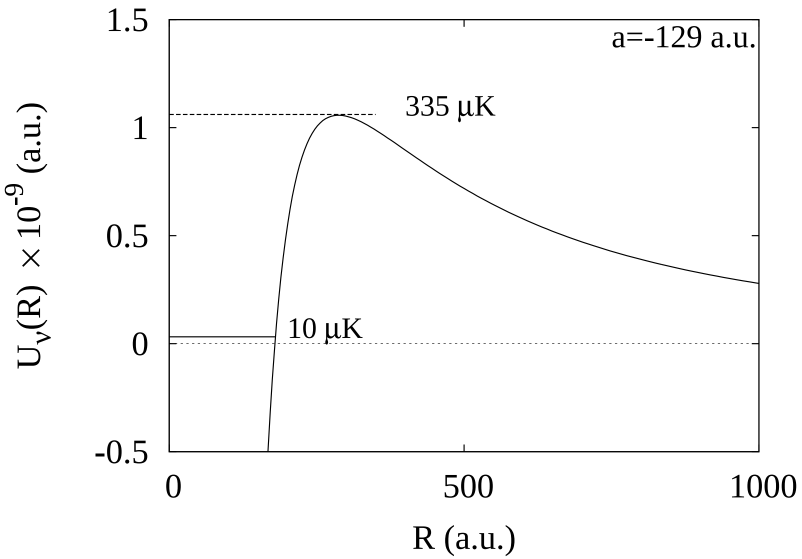
Three-Bosons System with $a < 0$

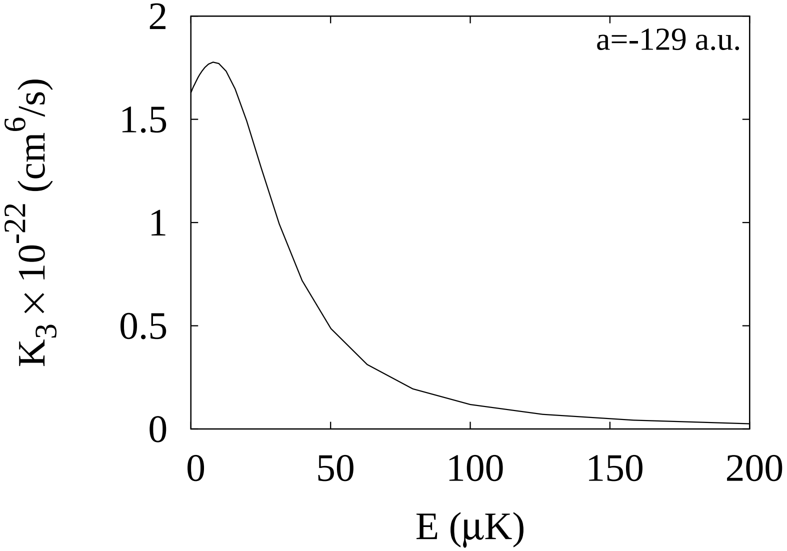
(Localized Coupling at $R \approx 50$ a.u.)

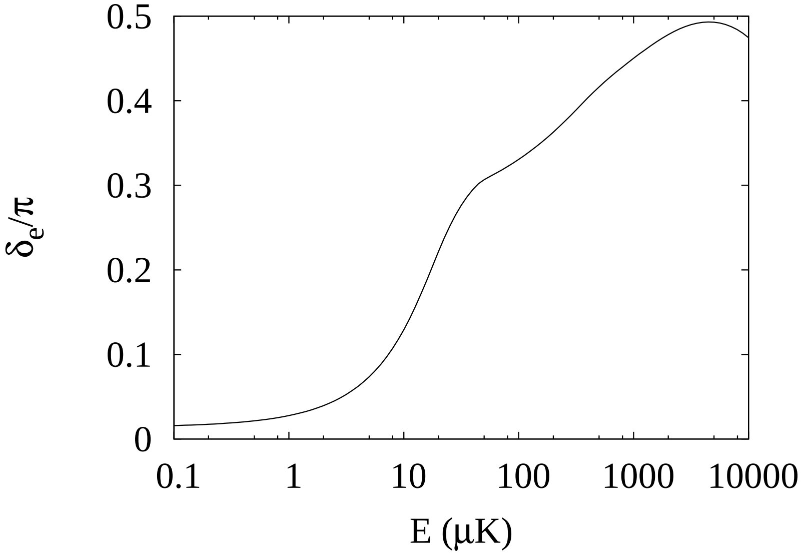
(Potential Barrier at $R \sim 2|a|$ $U_b = 0.079/\mu a^2$)

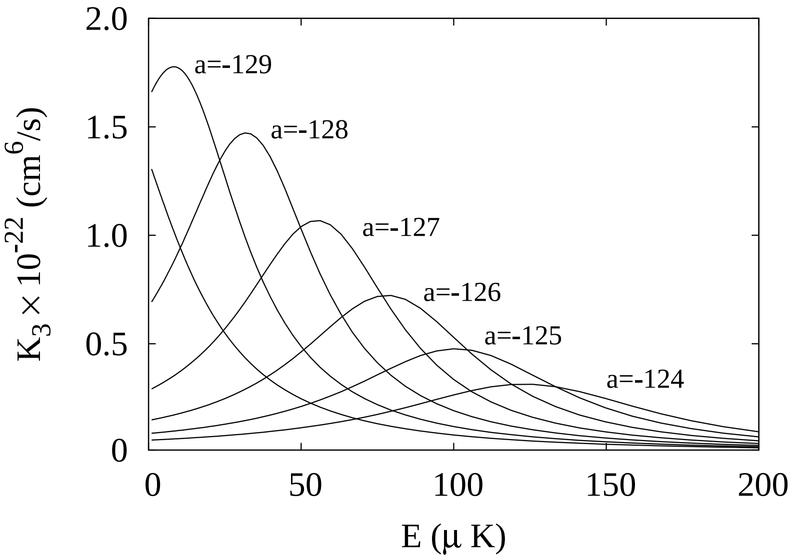


Recombination Rate for $a < 0$ (Resonance)

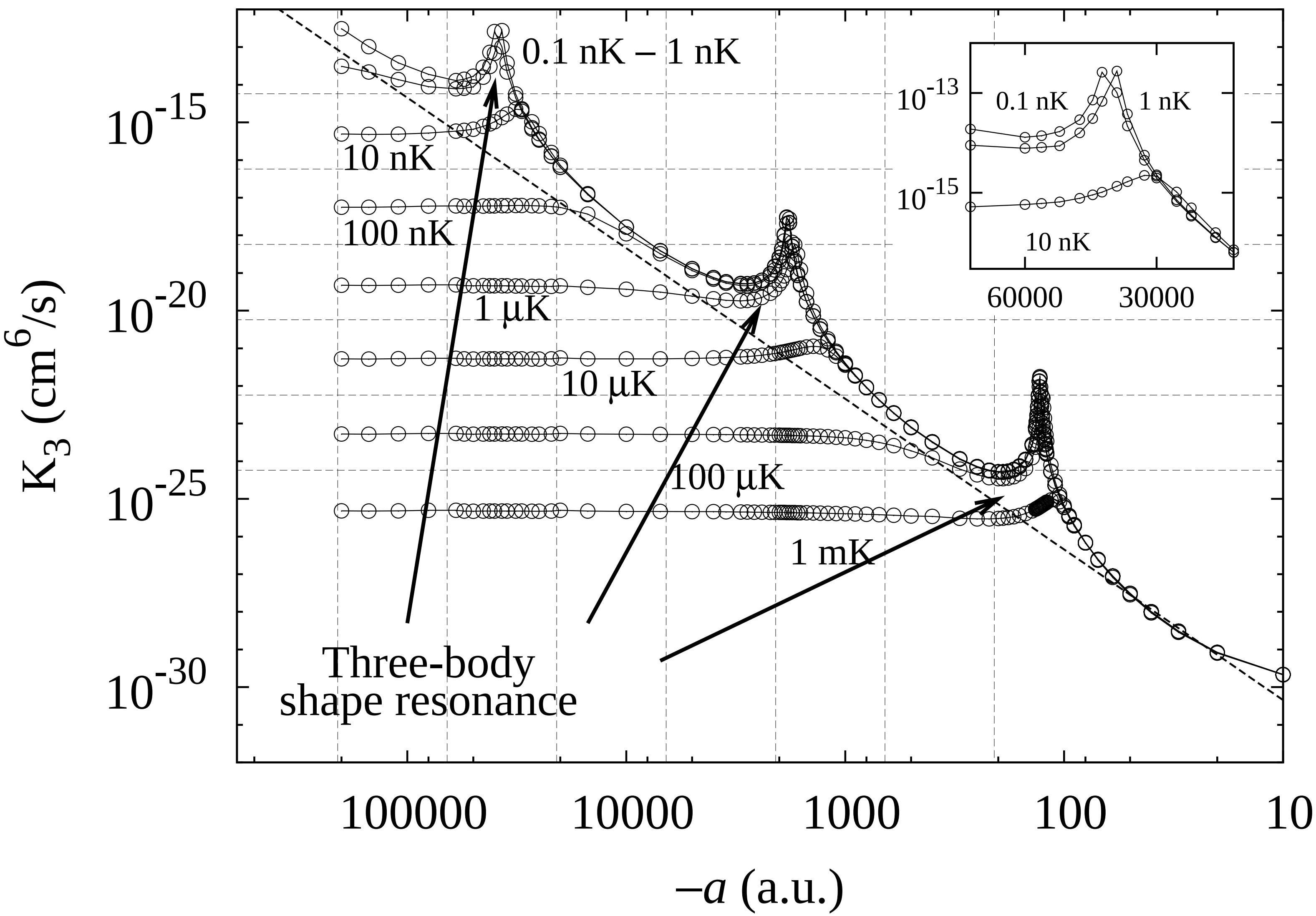








General Feature ($a < 0$)



Universal Behavior

- Previous works : $a \gg r_0$ / $kr_0 \ll 1$.
- Analytical Formula for $a > 0$: $(\alpha = 1.0064)$, $O(r_0/a)$

$$K_3 = 360\,(a^4/m)\sin^2\left[\alpha\ln(3a/2r_0) + \Phi\right]$$

- Analytical Formula for $a < 0$: $O(r_0/a)$

$$K_3 = \frac{4590\,(a^4/m)\sinh(2\eta_*)}{\sin^2[\alpha\ln(3|a|/2r_0) + \Phi + 1.63] + \sinh^2\eta_*}$$

- Minima and Peaks equally spaced $(e^{\pi/\alpha} \approx 22.7)$
- Efimov effect.

Threshold regime :

- $k|a| \lesssim 1$, or E is the smallest energy in the system.
- $a > 0$: $E_b = \hbar^2/ma^2$
- $a < 0$: Potential Barrier $(0.079\hbar^2/\mu a^2)$

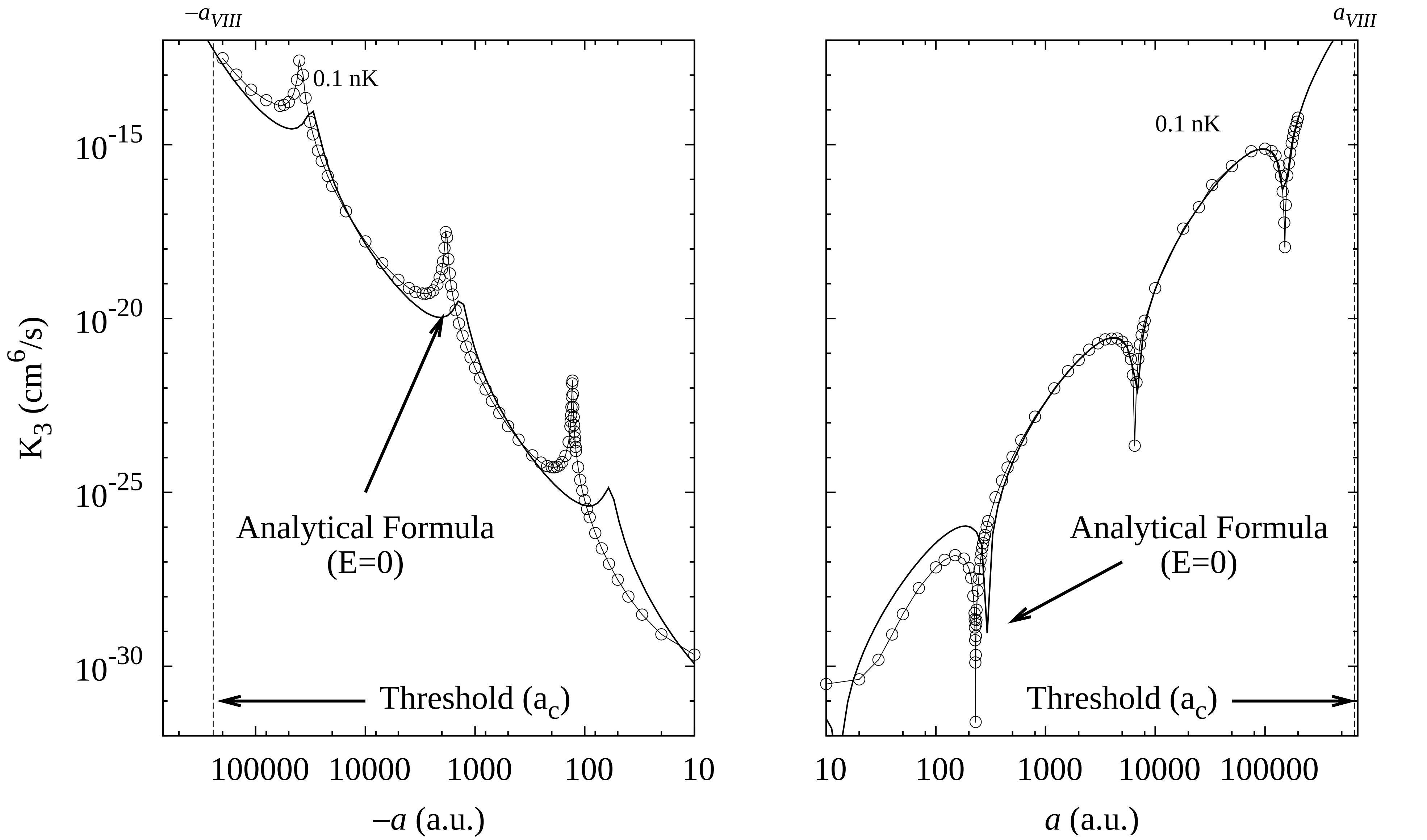
$(a > 0)$

$$E \lesssim \frac{\hbar^2}{ma^2} \Rightarrow a_c \lesssim \frac{\hbar}{\sqrt{mE}}$$

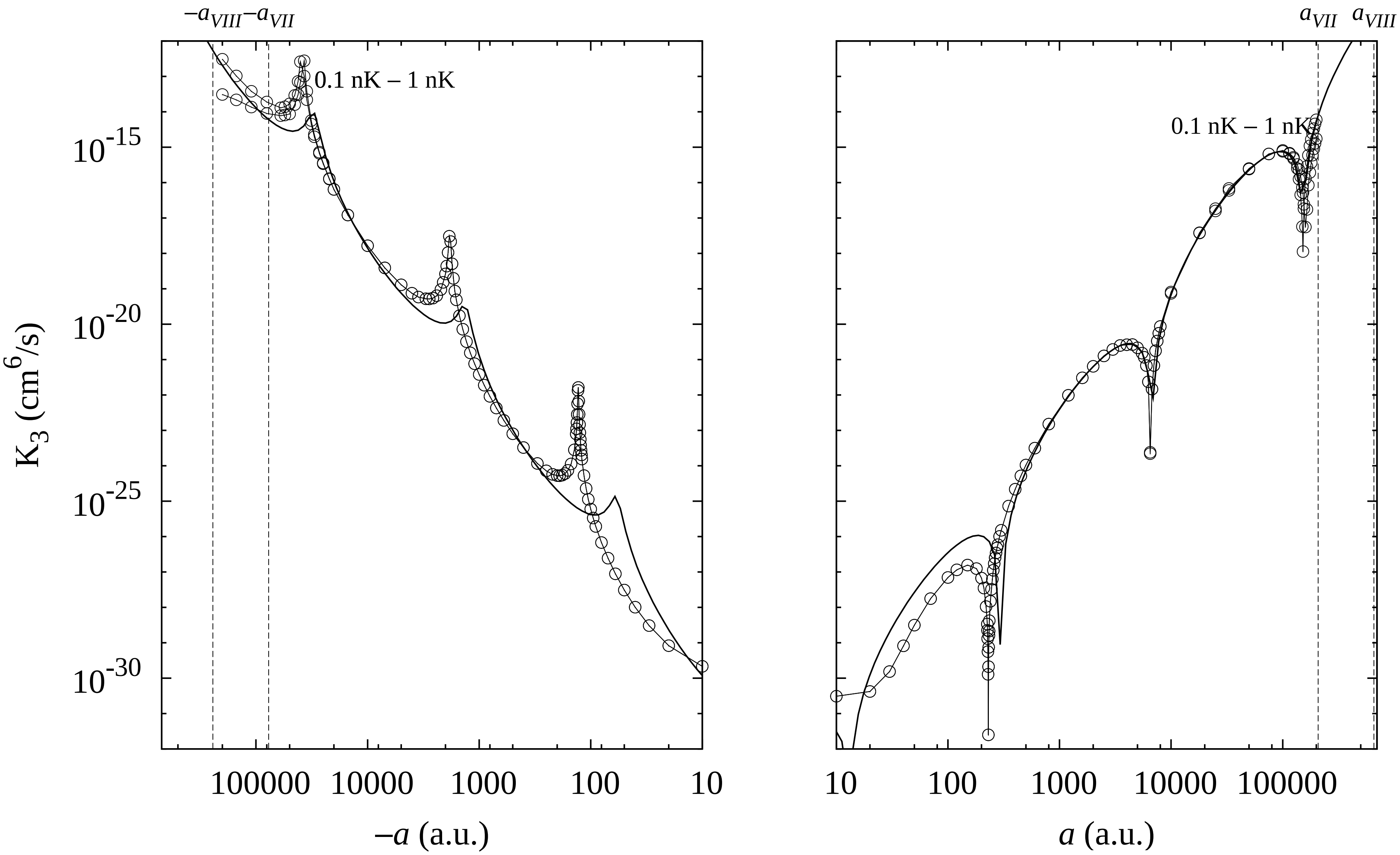
$(a < 0)$

$$E \lesssim \frac{0.079\hbar^2}{\mu a^2} \Rightarrow |a_c| \lesssim \frac{0.28\hbar}{\sqrt{\mu E}}$$

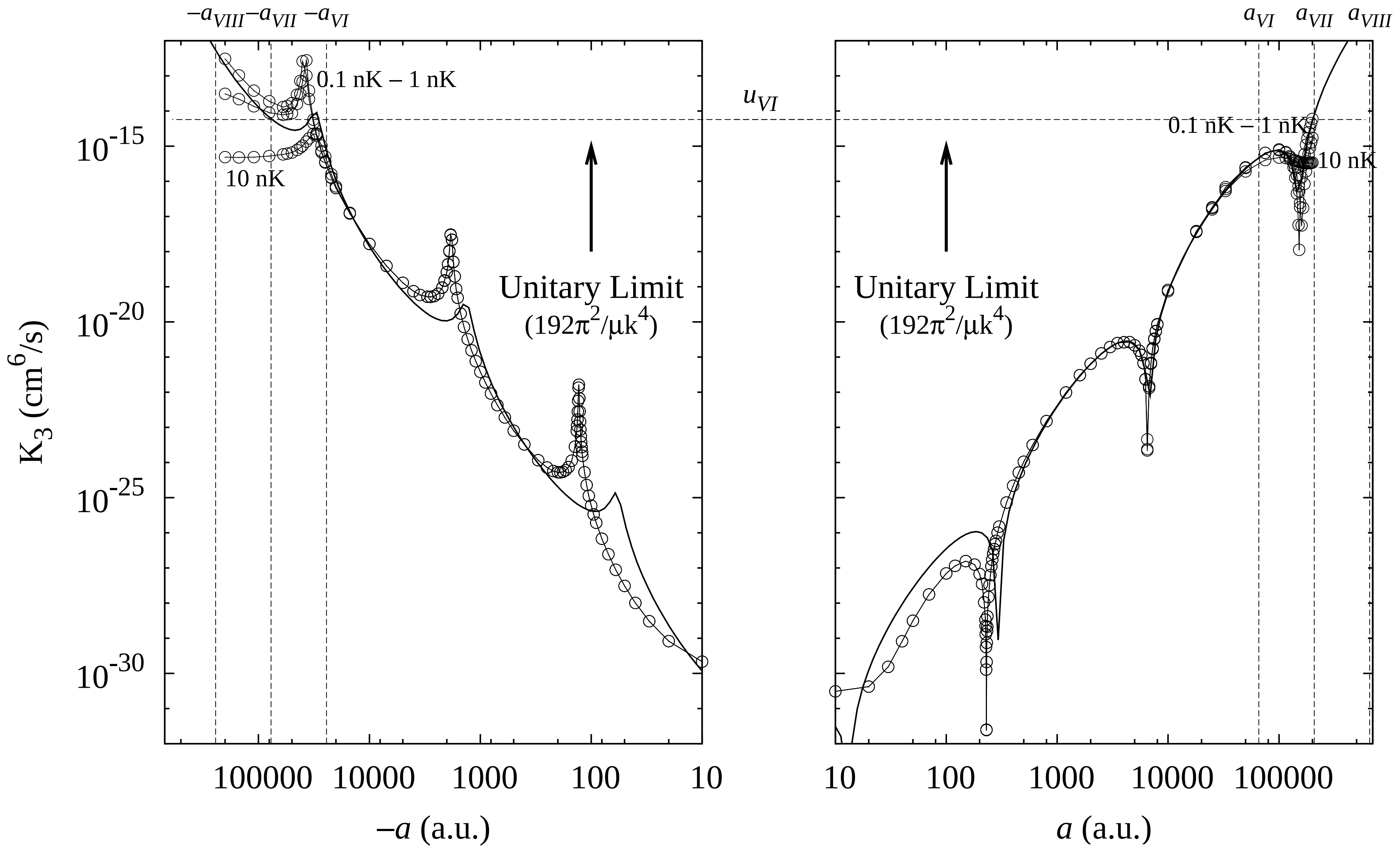
$$n \approx \frac{\pi}{\alpha} \ln\left(\frac{3a_c}{2r_0}\right)$$



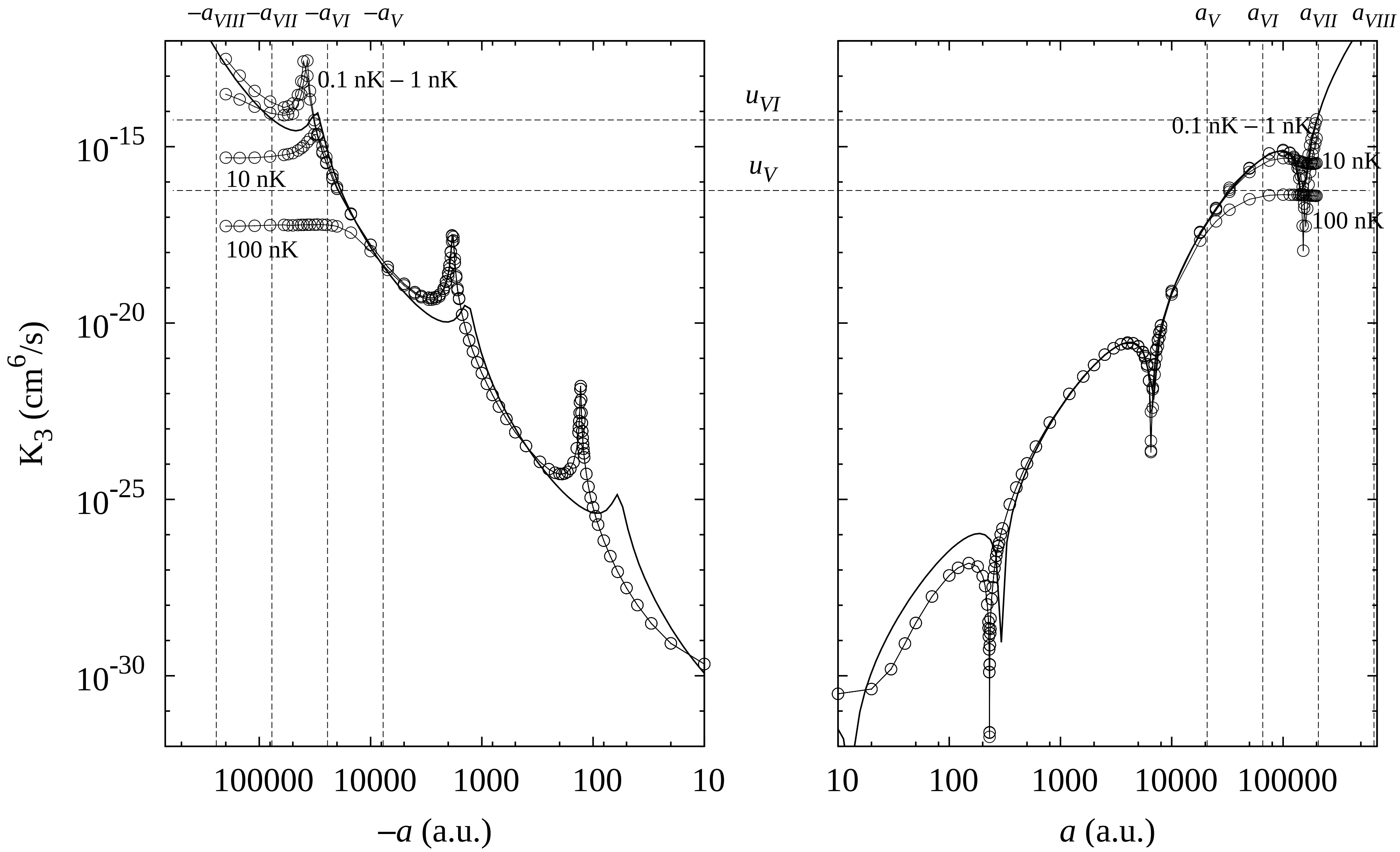
$-a_{VIII}$
a_{VIII}
0.1 nK
0.1 nK
10^{-15}
10^{-20}
10^{-25}
10^{-30}
K_3 (cm^6/s)
Analytical Formula (E=0)
Analytical Formula (E=0)
Threshold (a_c)
Threshold (a_c)
100000 10000 1000 100 10
10 100 1000 10000 100000
$-a$ (a.u.)
a (a.u.)



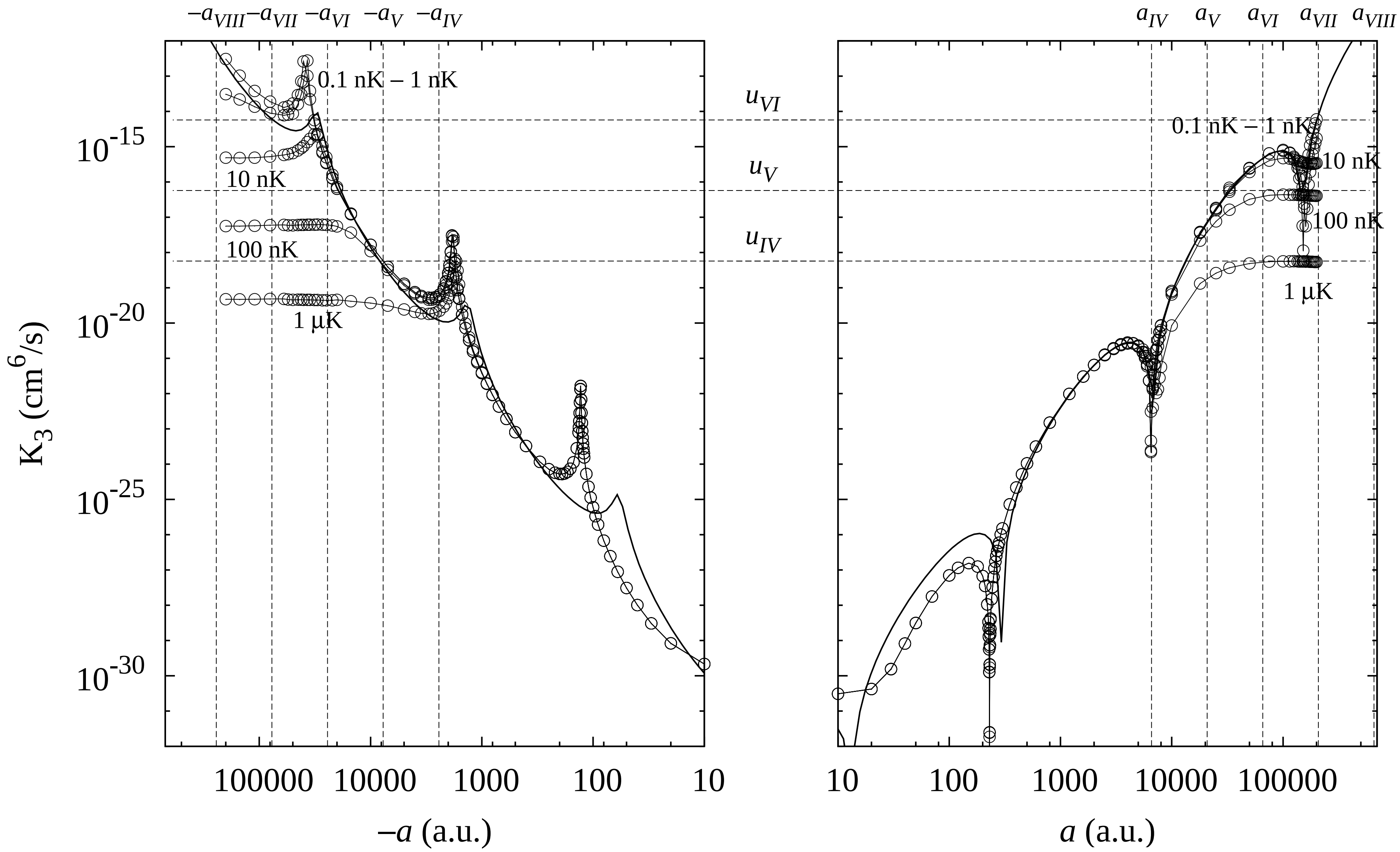
$-a_{VIII}$ $-a_{VII}$
0.1 nK – 1 nK
10^{-15}
10^{-20}
10^{-25}
10^{-30}
K_3 (cm^6/s)
100000 10000 1000 100 10
$-a$ (a.u.)
a_{VII} a_{VIII}
0.1 nK – 1 nK
10 100 1000 10000 100000
a (a.u.)



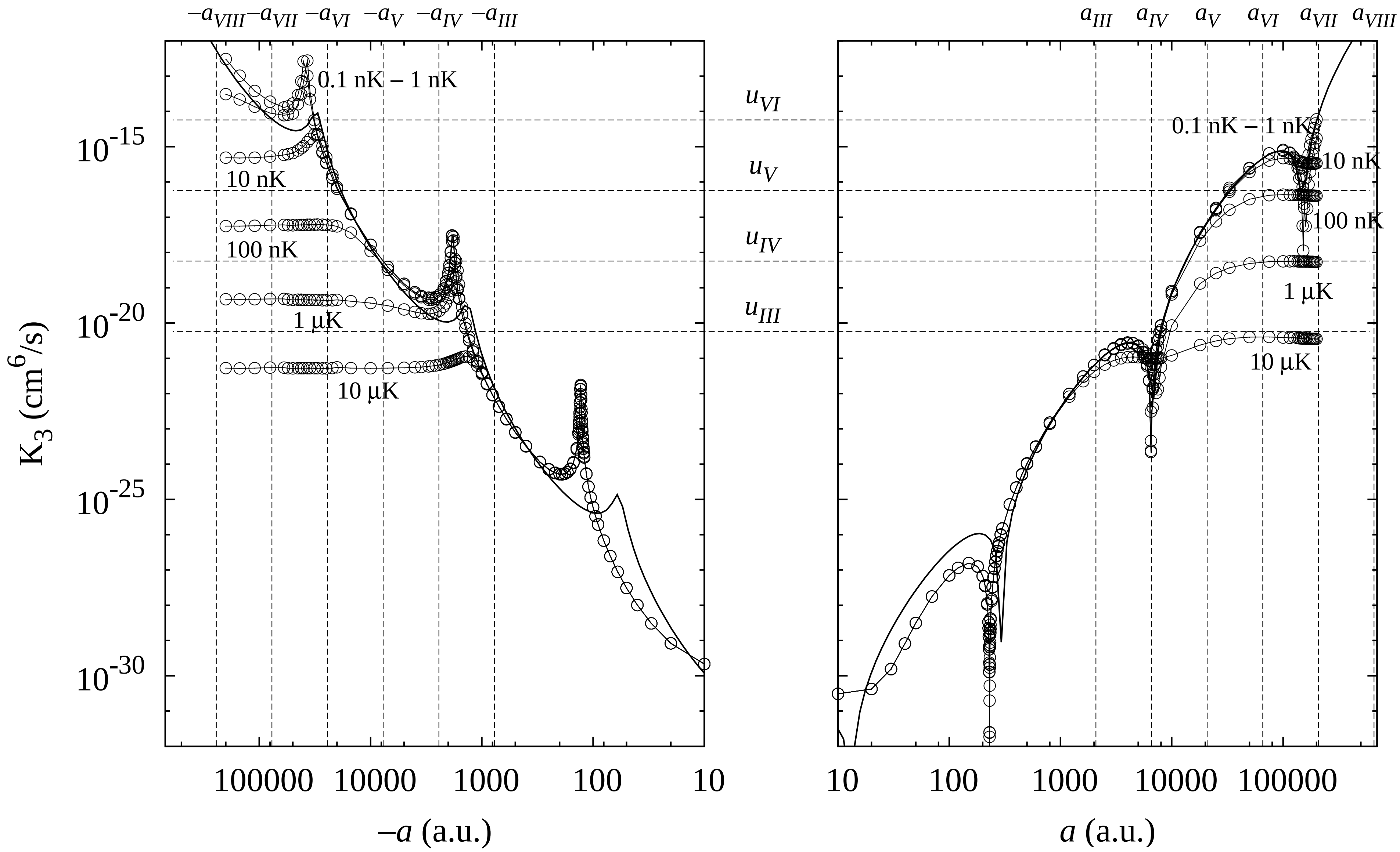
$-a_{VIII}$ $-a_{VII}$ $-a_{VI}$
a_{VI} a_{VII} a_{VIII}
u_{VI}
0.1 nK – 1 nK
10 nK
0.1 nK – 1 nK
10 nK
Unitary Limit
($192\pi^2/\mu k^4$)
Unitary Limit
($192\pi^2/\mu k^4$)
K_3 (cm^6/s)
10^{-15}
10^{-20}
10^{-25}
10^{-30}
100000 10000 1000 100 10
10 100 1000 10000 100000
$-a$ (a.u.)
a (a.u.)



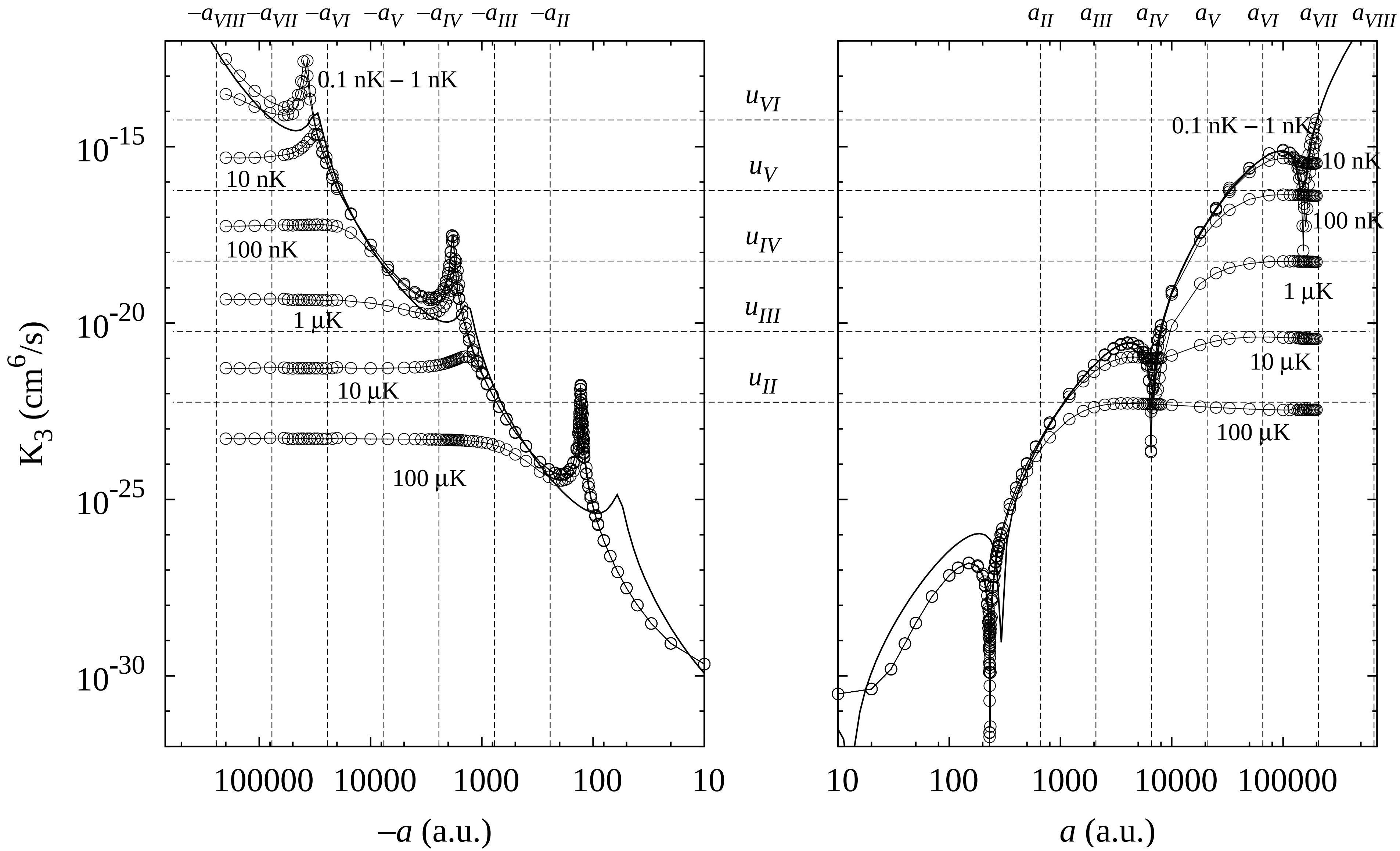
$-a_{VIII}$ $-a_{VII}$ $-a_{VI}$ $-a_V$
a_V a_{VI} a_{VII} a_{VIII}
u_{VI}
u_V
0.1 nK – 1 nK
10 nK
100 nK
0.1 nK – 1 nK
10 nK
100 nK
10^{-15}
10^{-20}
10^{-25}
10^{-30}
K_3 (cm^6/s)
100000
10000
1000
100
10
10
100
1000
10000
100000
$-a$ (a.u.)
a (a.u.)



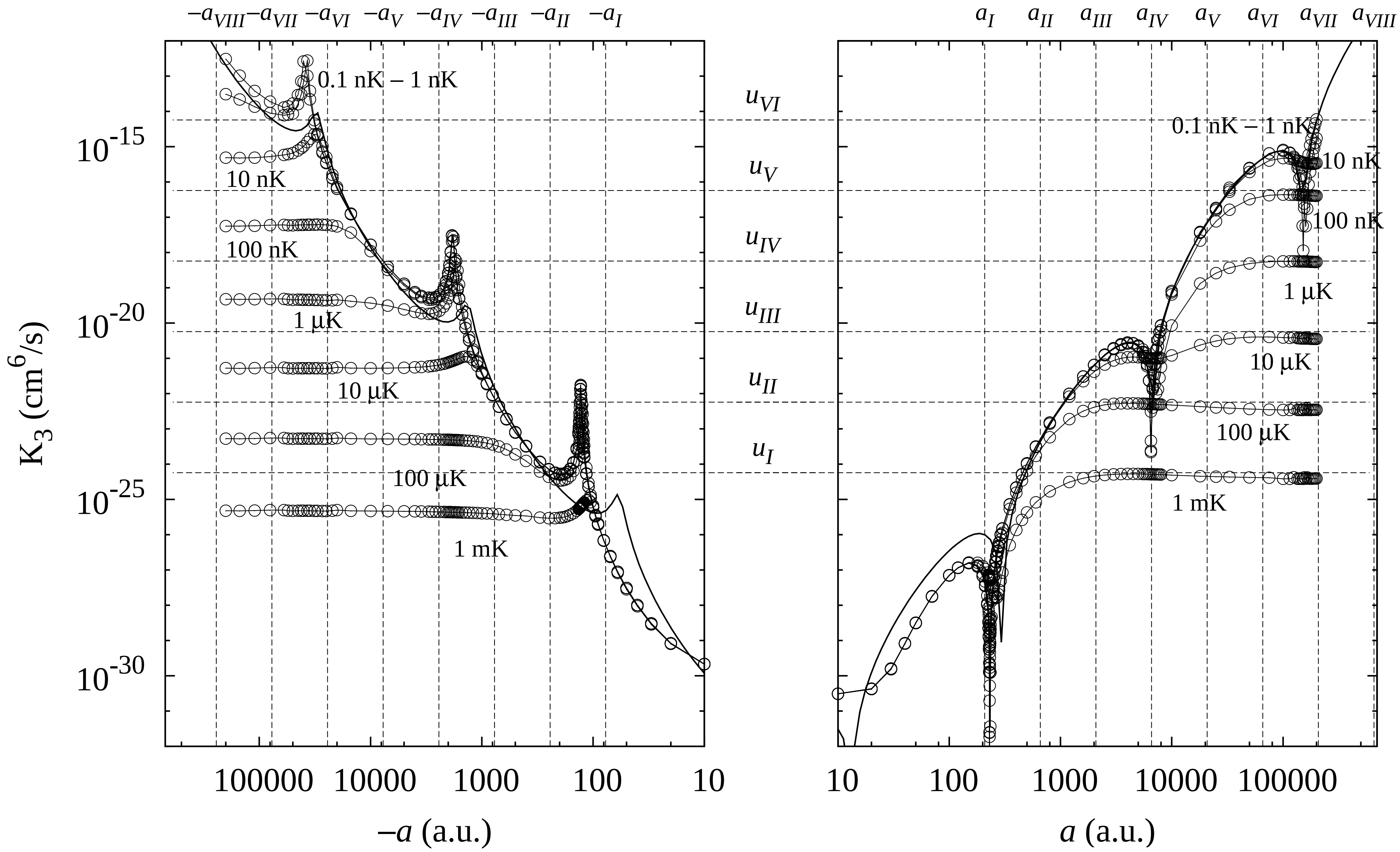
$-a_{VIII}$ $-a_{VII}$ $-a_{VI}$ $-a_V$ $-a_{IV}$
a_{IV} a_V a_{VI} a_{VII} a_{VIII}
u_{VI}
u_V
u_{IV}
0.1 nK – 1 nK
10 nK
100 nK
1 μK
0.1 nK – 1 nK
10 nK
100 nK
1 μK
K_3 (cm^6/s)
10^{-15}
10^{-20}
10^{-25}
10^{-30}
100000 10000 1000 100 10
10 100 1000 10000 100000
$-a$ (a.u.)
a (a.u.)



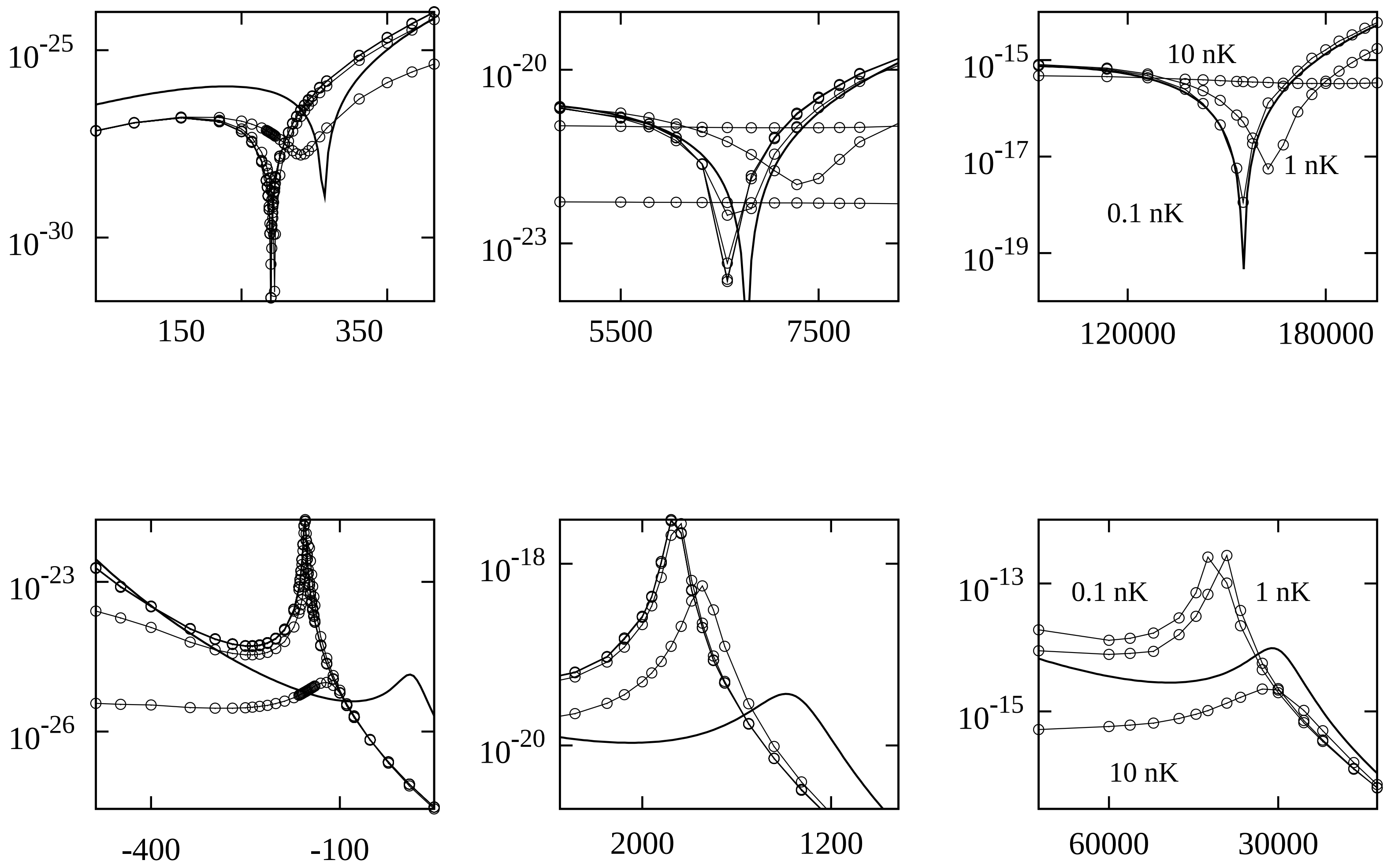
$-a_{VIII}$ $-a_{VII}$ $-a_{VI}$ $-a_{V}$ $-a_{IV}$ $-a_{III}$
a_{III} a_{IV} a_{V} a_{VI} a_{VII} a_{VIII}
u_{VI}
u_{V}
u_{IV}
u_{III}
0.1 nK – 1 nK
10 nK
100 nK
1 μK
10 μK
10^{-15}
10^{-20}
10^{-25}
10^{-30}
K_3 (cm^6/s)
100000 10000 1000 100 10
10 100 1000 10000 100000
$-a$ (a.u.)
a (a.u.)



−a_VIII −a_VII −a_VI −a_V −a_IV −a_III −a_II
a_II a_III a_IV a_V a_VI a_VII a_VIII
u_VI
u_V
u_IV
u_III
u_II
0.1 nK – 1 nK
10 nK
100 nK
1 μK
10 μK
100 μK
10^-15
10^-20
10^-25
10^-30
K_3 (cm^6/s)
100000
10000
1000
100
10
−a (a.u.)
a (a.u.)



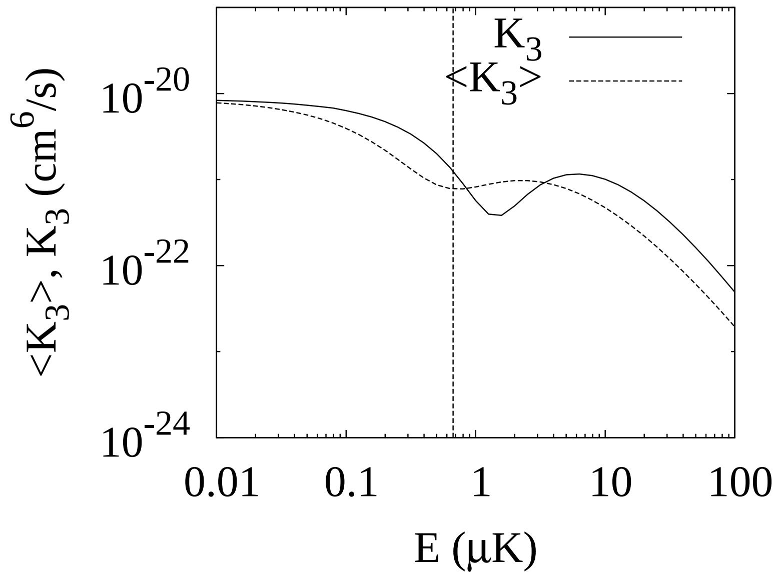
−a_VIII −a_VII −a_VI −a_V −a_IV −a_III −a_II −a_I
a_I a_II a_III a_IV a_V a_VI a_VII a_VIII
u_VI
u_V
u_IV
u_III
u_II
u_I
0.1 nK − 1 nK
10 nK
100 nK
1 μK
10 μK
100 μK
1 mK
K_3 (cm^6/s)
10^-15
10^-20
10^-25
10^-30
100000 10000 1000 100 10
10 100 1000 10000 100000
−a (a.u.)
a (a.u.)

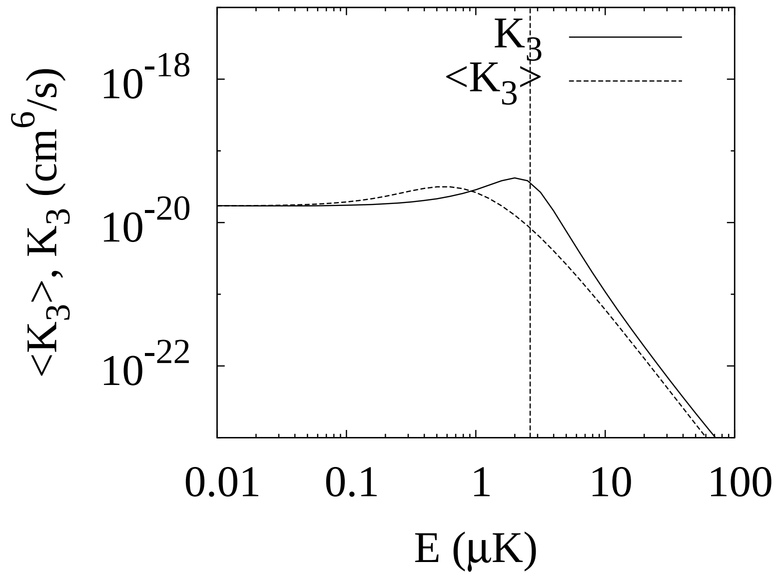


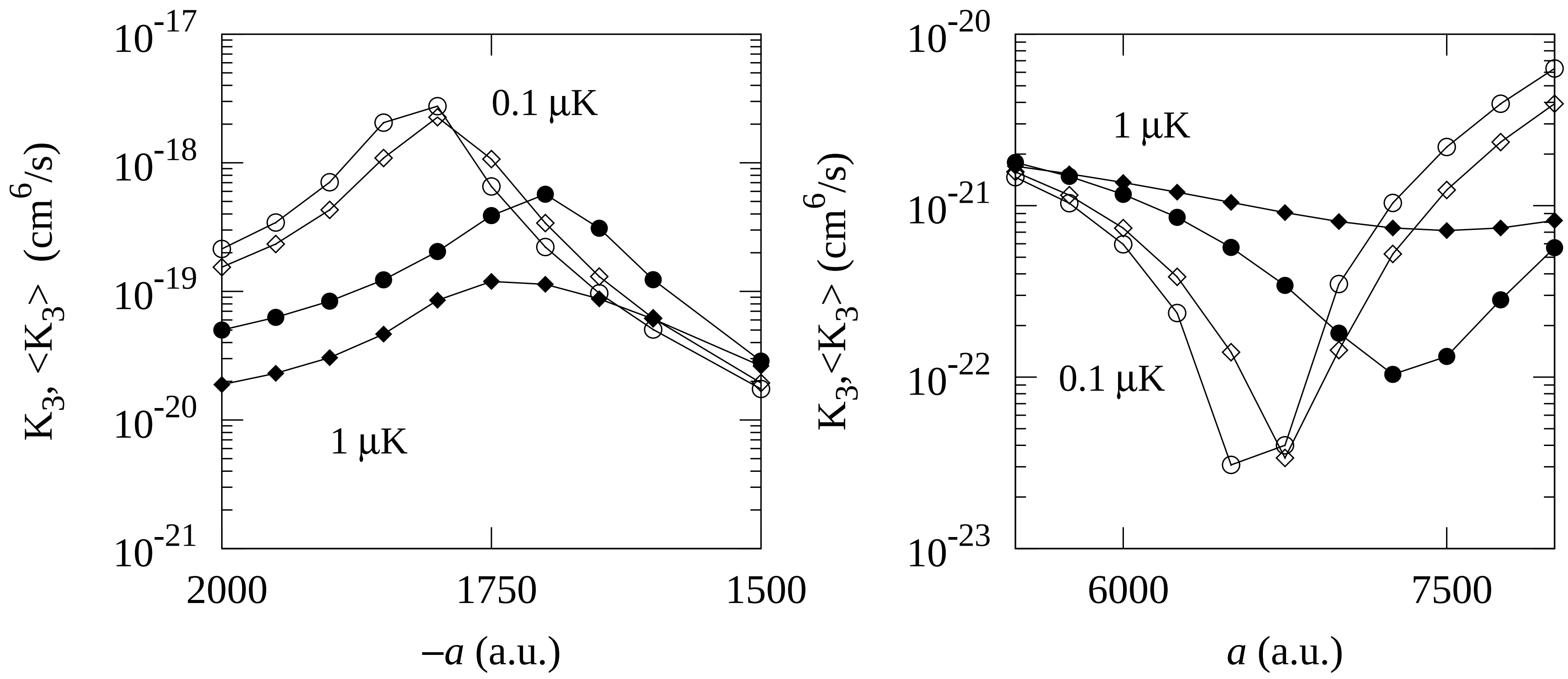
10-25
10-30
150
350
10-20
10-23
5500
7500
10 nK
10-15
10-17
1 nK
0.1 nK
10-19
120000
180000
10-23
10-26
-400
-100
10-18
10-20
2000
1200
10-13
0.1 nK
1 nK
10-15
10 nK
60000
30000

Thermal Average

$$\langle K_3\rangle(T) = \frac{1}{2(k_BT)^3}\int K_3(E)E^2 e^{-E/k_BT}dE$$

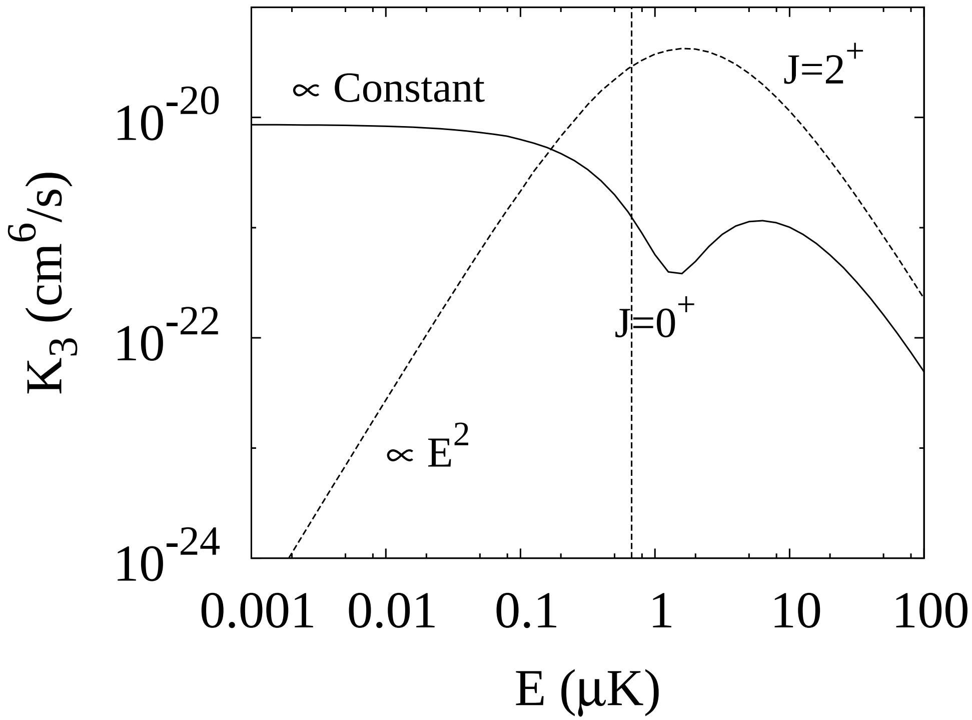


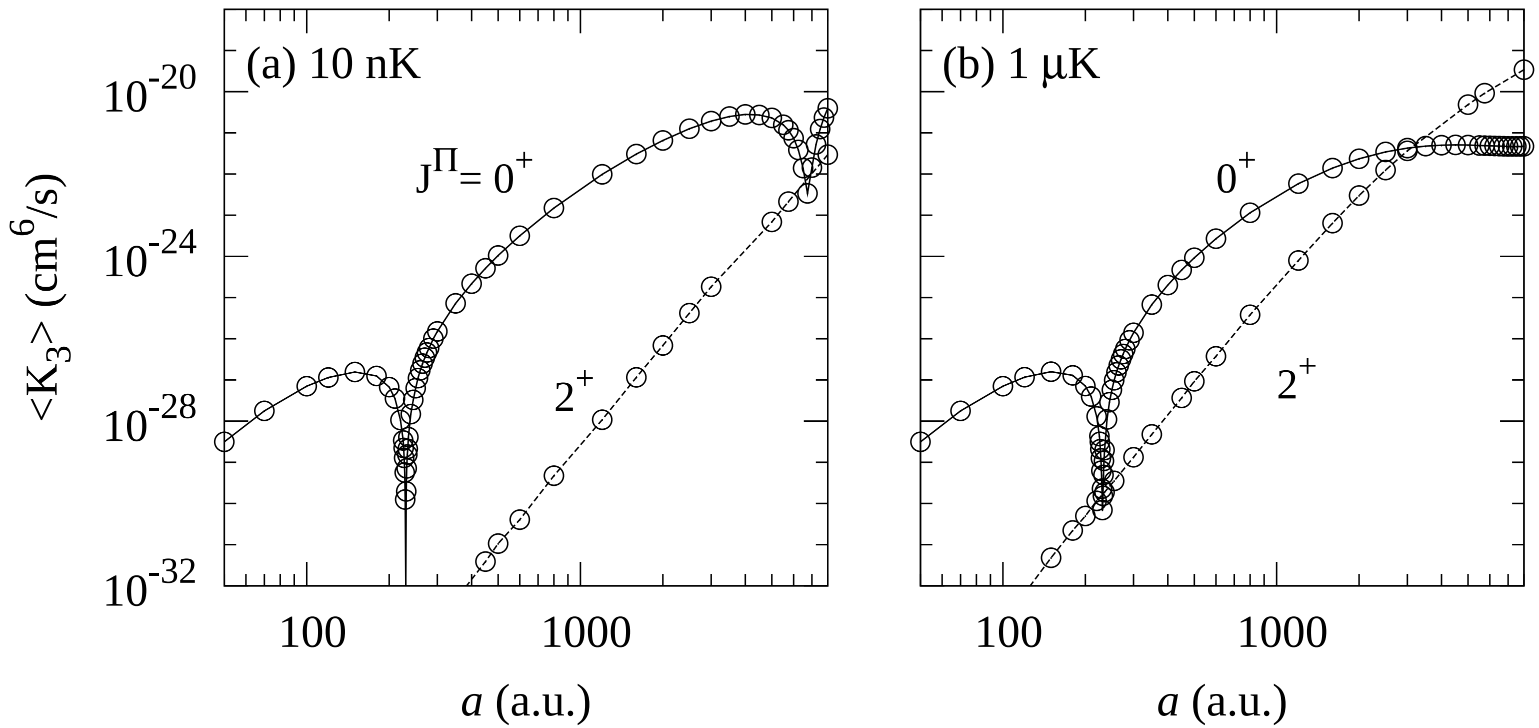




10-17
10-18
10-19
10-20
10-21
0.1 μK
1 μK
K3, <K3> (cm6/s)
2000
1750
1500
−a (a.u.)
10-20
10-21
10-22
10-23
1 μK
0.1 μK
6000
7500
a (a.u.)

Hight Partial Waves




(a) 10 nK
(b) 1 μK
J^Π = 0^+
0^+
2^+
2^+
10^-20
10^-24
10^-28
10^-32
100
1000
100
1000
<K_3> (cm^6/s)
a (a.u.)
a (a.u.)

Conclusions

- Universal Behavior

$(a > 0)$

$$E \lesssim \frac{\hbar^2}{ma^2} \Rightarrow a_c \lesssim \frac{\hbar}{\sqrt{mE}}$$

$(a < 0)$

$$E \lesssim \frac{0.079\hbar^2}{\mu a^2} \Rightarrow |a_c| \lesssim \frac{0.28\hbar}{\sqrt{\mu E}}$$

$$n \approx \frac{\pi}{\alpha} \ln\left(\frac{3a_c}{2r_0}\right)$$

- ^{4}He : 100 nK $\Rightarrow -7700$ a.u. $< a <$ 20800 a.u. $\Rightarrow n \approx 2$
- ^{23}Na : 100 nK $\Rightarrow -3200$ a.u. $< a <$ 8650 a.u. $\Rightarrow n \approx 2$
- ^{87}Rb : 100 nK $\Rightarrow -1650$ a.u. $< a <$ 4450 a.u. $\Rightarrow n \approx 2$

- Efimov Effect, $n = 3$: ^{4}He $\Rightarrow$ 3 nK / ^{23}Na $\Rightarrow$ 0.5 nK / ^{87}Rb $\Rightarrow$ 0.1 nK
- $n = 4$: ^{4}He $\Rightarrow$ 6. 10^{-3} nK / ^{23}Na $\Rightarrow$ 1. 10^{-3} nK / ^{87}Rb $\Rightarrow$ 3. 10^{-4} nK

Future :

- More Diatomic channels : Universal Behaviour

- Relaxation Process : $X + X_2^* \rightarrow X + X_2$

- Fermions : Molecular BEC / Cooper Pairs !